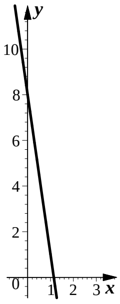
Aufgabe 12.106

Bestimmen Sie die Monotoniebereiche, Extrema und Wertebereiche folgender Funktionen:

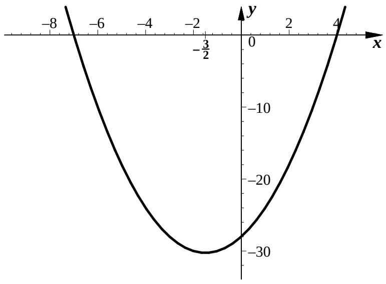
a) $y(x) = 8 - 7x$,
b) $y(x) = x^2 + 3x - 28$,
c) $y(x) = x^3 + 27$,
d) $y(x) = x^3 - 27x$!



Lösung:

a) $x = 0 \;\Rightarrow\; y = 8, \quad y = 0 \;\Rightarrow\; x = \dfrac{8}{7}$

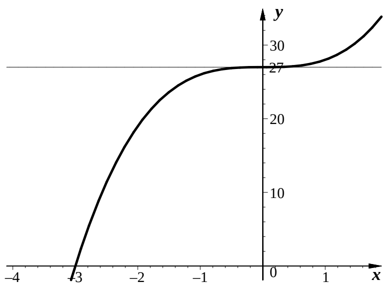
$y' = -7 < 0$, überall monoton fallend, keine Extrema, WB $= \mathbb{R}$



b) $$y' = 2x+3 \begin{cases} <0 & x<-3/2 \quad \text{monoton fallend} \\ =0 & x=-3/2 \quad \Longrightarrow \text{ Minimum} \\ >0 & x>-3/2 \quad \text{monoton wachsend} \end{cases}$$

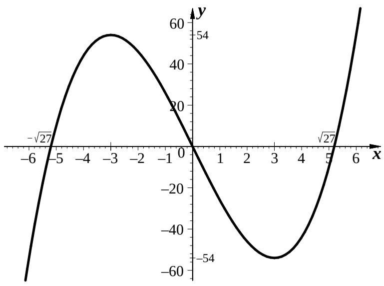
$$\text{Min.: } y\left(-\frac{3}{2}\right) = \frac{9}{4} - \frac{9}{2} - 28 = \frac{9-18-112}{4} = -\frac{121}{4}$$

$$\Longrightarrow \text{WB} = \left\{ y : y \geq -\frac{121}{4} \right\}$$



c) $$y' = 3x^2 \begin{cases} >0 & x<0 \quad \text{monoton wachsend} \\ =0 & x=0 \quad \Longrightarrow \text{ kein Extremum: Sattelpunkt} \\ >0 & x>0 \quad \text{monoton wachsend} \end{cases}$$

WB $= \mathbb{R}$



d) $y' = 3x^2 - 27 = 3(x^2-9) = 3(x+3)(x-3)$,

$$y' = \begin{cases} >0 & x<-3 \quad \text{monoton wachsend} \\ =0 & x=-3 \quad \Longrightarrow \text{ Maximum } y(-3)=54 \\ <0 & -3<x<3 \quad \text{monoton fallend} \\ =0 & x=3 \quad \Longrightarrow \text{ Minimum } y(3)=-54 \\ >0 & x>3 \quad \text{monoton wachsend} \end{cases}$$

WB $= \mathbb{R}$