Aufgabe 12.75

Berechnen Sie folgende Grenzwerte:

a) $\lim\limits_{x\to 1}\dfrac{x^3-1}{\ln x}$, b) $\lim\limits_{x\to 0}\dfrac{e^{3x}-1}{\ln(1+x)}$, c) $\lim\limits_{x\to 1}\dfrac{\cos\frac{\pi}{2}x}{x-1}$, d) $\lim\limits_{x\to 0}\dfrac{1-\cos x}{2x^2}$, e) $\lim\limits_{x\to 0}\dfrac{e^{2x}-1}{\ln(1+2x)}$,

f) $\lim\limits_{x\to\infty}\dfrac{\frac{\pi}{2}-\arctan x}{\ln\left(1+\frac{1}{x^2}\right)}$, g) $\lim\limits_{x\to 0}(1-e^{2x})\cot x$!

Lösung:

a) $\dfrac{0}{0}:\quad \lim\limits_{x\to 1}\dfrac{x^3-1}{\ln x}=\lim\limits_{x\to 1}\dfrac{3x^2}{\frac{1}{x}}=3$

b) $\dfrac{0}{0}:\quad \lim\limits_{x\to 0}\dfrac{e^{3x}-1}{\ln(1+x)}=\lim\limits_{x\to 0}\dfrac{3e^{3x}}{\frac{1}{1+x}}=3$

c) $\dfrac{0}{0}:\quad \lim\limits_{x\to 1}\dfrac{\cos\frac{\pi}{2}x}{x-1}=\lim\limits_{x\to 1}\dfrac{-\frac{\pi}{2}\sin\frac{\pi}{2}x}{1}=-\dfrac{\pi}{2}$

d) $\dfrac{0}{0}:\quad \lim\limits_{x\to 0}\dfrac{1-\cos x}{2x^2}=\lim\limits_{x\to 0}\dfrac{\sin x}{4x}=\lim\limits_{x\to 0}\dfrac{\cos x}{4}=\dfrac{1}{4}$

e) $\dfrac{0}{0}:\quad \lim\limits_{x\to 0}\dfrac{e^{2x}-1}{\ln(1+2x)}=\lim\limits_{x\to 0}\dfrac{2e^{2x}}{\frac{2}{1+2x}}=1$

f) $\dfrac{0}{0}:\quad \lim\limits_{x\to\infty}\dfrac{\frac{\pi}{2}-\arctan x}{\ln\left(1+\frac{1}{x^2}\right)}=\lim\limits_{x\to\infty}\dfrac{-\frac{1}{1+x^2}}{\frac{1}{1+\frac{1}{x^2}}(-\frac{2}{x^3})}=\lim\limits_{x\to\infty}\dfrac{x}{2}=\infty$

g) $0\cdot\infty:\quad \lim\limits_{x\to 0}(1-e^{2x})\cot x=\lim\limits_{x\to 0}\dfrac{(1-e^{2x})}{\tan x}=\lim\limits_{x\to 0}\dfrac{-2e^{2x}}{\frac{1}{\cos^2 x}}=-2$