Aufgabe 12.41

Berechnen Sie die ersten Ableitungen folgender Funktionen:

a) $f(x) = x^5\, 5^x$, b) $f(x) = \left(x+\sin^2 x+\cos^2 x\right)^5$, c) $f(x) = \ln\sqrt{x^2+\sin^2 x}$,

d) $f(x) = \cos(x^2+2)\,(3x-4)^3\,\ln(3x-4)$, e) $f(x) = \dfrac{(x^2+3x+5)\sin x}{x\cos x}$!

Lösung:

a) $f'(x) = 5x^4 5^x + x^5 5^x \ln 5 = x^4 5^x (5+x\ln 5)$

b) $f(x) = (x+1)^5,\quad f'(x) = 5(x+1)^4$

c) $f'(x) = \dfrac{1}{\sqrt{x^2+\sin^2 x}}\,\dfrac{2x+2\sin x\cos x}{2\sqrt{x^2+\sin^2 x}} = \dfrac{x+\sin x\cos x}{x^2+\sin^2 x}$

d)
$$\begin{aligned} f'(x) &= -\sin(x^2+2)\,2x\,(3x-4)^3\ln(3x-4) + \cos(x^2+2)\,3\cdot 3\,(3x-4)^2\ln(3x-4) \\ &\quad + \cos(x^2+2)\,(3x-4)^3\,\frac{3}{3x-4} \\ &= -2x\sin(x^2+2)\,(3x-4)^3\ln(3x-4) + \big((9\ln(3x-4)+3\big)\cos(x^2+2)\,(3x-4)^2 \end{aligned}$$

e)
$$\begin{aligned} f'(x) &= \frac{\big((2x+3)\sin x + (x^2+3x+5)\cos x\big)x\cos x - (x^2+3x+5)\sin x(\cos x - x\sin x)}{x^2\cos^2 x} \\ &= \frac{(2x+3)\,x\sin x\cos x + (x^2+3x+5)\,(x\cos^2 x - \sin x\cos x + x\sin^2 x)}{x^2\cos^2 x} \\ &= \frac{(2x^2+3x)\,\sin x\cos x + (x^2+3x+5)\,(x-\sin x\cos x)}{x^2\cos^2 x} \\ &= \frac{(2x^2+3x-x^2-3x-5)\,\sin x\cos x + x^3+3x^2+5x}{x^2\cos^2 x} \\ &= \frac{(x^2-5)\,\sin x\cos x + x^3+3x^2+5x}{x^2\cos^2 x} \end{aligned}$$