

Aufgabe 12.40

Berechnen Sie die ersten Ableitungen folgender Funktionen:

a) $f(x) = (4x^3 + 2x) (\sin 3x + 2) \sin(3x + 2)$, b) $f(x) = (x \ln x)^5$, c) $f(x) = \ln \sqrt{x\sqrt{x}}$,
d) $f(x) = \sin^2(x^2 + 1) + \cos^2(x^2 + 1) + \sin(x^2 + 1)^2 + \cos(x^2 + 1)^2$, e) $f(x) = \frac{x \cos(a - bx)}{x^2 + 1}$!

Lösung:

a) $f'(x) = (12x^2 + 2) (\sin 3x + 2) \sin(3x + 2) + 3(4x^3 + 2x) \cos 3x \sin(3x + 2) + 3(4x^3 + 2x) (\sin 3x + 2) \cos(3x + 2)$

b) $f'(x) = 5(x \ln x)^4 \left(\ln x + \frac{x}{x} \right) = 5(x \ln x)^4 (\ln x + 1)$

c) $f(x) = \ln \sqrt{x x^{1/2}} = \ln \sqrt{x^{3/2}} = \ln x^{3/4}$, $f'(x) = \frac{1}{x^{3/4}} \frac{3}{4} x^{-1/4} = \frac{3}{4x}$

oder $f'(x) = \frac{1}{\sqrt{x\sqrt{x}}} \frac{1}{2\sqrt{x\sqrt{x}}} \left(\sqrt{x} + \frac{x}{2\sqrt{x}} \right) = \frac{1}{2x\sqrt{x}} \left(\sqrt{x} + \frac{\sqrt{x}}{2} \right) = \frac{1}{2x\sqrt{x}} \frac{3}{2} \sqrt{x} = \frac{3}{4x}$

d) $f(x) = 1 + \sin(x^2 + 1)^2 + \cos(x^2 + 1)^2$,
 $f'(x) = 2(x^2 + 1) 2x \cos(x^2 + 1)^2 - 2(x^2 + 1) 2x \sin(x^2 + 1)^2 = 4(x^2 + 1) \left(\cos(x^2 + 1)^2 - \sin(x^2 + 1)^2 \right)$

e) $f'(x) = \frac{(\cos(a - bx) + bx \sin(a - bx)) (x^2 + 1) - x \cos(a - bx) 2x}{(x^2 + 1)^2}$
 $= \frac{(1 - x^2) \cos(a - bx) + bx(x^2 + 1) \sin(a - bx)}{(x^2 + 1)^2} = \frac{(1 - x^2) \cos(a - bx)}{(1 + x^2)^2} + \frac{bx \sin(a - bx)}{1 + x^2}$