Aufgabe 12.39

Berechnen Sie die ersten Ableitungen folgender Funktionen:

a) $f(x)=(3-2x)\sin\left((3-2x)^2\right)\left(\sin(3-2x)\right)^2$, b) $f(x)=(\mathrm{e}^x\ln x)^2$, c) $f(x)=\ln\sqrt{\dfrac{x^3}{\sqrt{x}}}$,

d) $f(x)=\sin(3-x^2)^2+\cos(3-x^2)^2+\sin^2(3-x^2)+\cos^2(3-x^2)$, e) $f(x)=\dfrac{x\sin(ax+b)}{x^2+3}$!

Lösung:

a)
$$\begin{aligned} f'(x) &= -2\sin(3-2x)^2\left(\sin(3-2x)\right)^2+(3-2x)\cos(3-2x)^2\,2(3-2x)(-2)\left(\sin(3-2x)\right)^2 \\ &\quad +(3-2x)\sin(3-2x)^2\,2\sin(3-2x)\cos(3-2x)\,(-2) \\ &= -2\sin(3-2x)^2\left(\sin(3-2x)\right)^2-4\,(3-2x)^2\cos(3-2x)^2\left(\sin(3-2x)\right)^2 \\ &\quad -4\,(3-2x)\sin(3-2x)^2\sin(3-2x)\cos(3-2x) \end{aligned}$$

b)
$$f'(x)=2\,\mathrm{e}^x\ln x\left(\mathrm{e}^x\ln x+\frac{\mathrm{e}^x}{x}\right)=2\,\mathrm{e}^{2x}\ln x\left(\ln x+\frac{1}{x}\right)$$

c)
$$f(x)=\ln\sqrt{x^{5/2}}=\ln x^{5/4},\quad f'(x)=\frac{1}{x^{5/4}}\,\frac{5}{4}x^{1/4}=\frac{5}{4x}$$

oder
$$f'(x)=\frac{1}{\sqrt{\dfrac{x^3}{\sqrt{x}}}}\,\frac{1}{2\sqrt{\dfrac{x^3}{\sqrt{x}}}}\,\frac{3x^2\sqrt{x}-x^3\dfrac{1}{2\sqrt{x}}}{x}=\frac{\sqrt{x}}{2x^3}\,\frac{6x^3-x^3}{2x\sqrt{x}}=\frac{5}{4x}$$

d)
$$f(x)=\sin(3-x^2)^2+\cos(3-x^2)^2+1,$$
$$\begin{aligned} f'(x) &= \cos(3-x^2)^2\cdot 2\,(3-x^2)(-2x)-\sin(3-x^2)^2\cdot 2\,(3-x^2)(-2x) \\ &= 4x\,(3-x^2)^2\left(\sin(3-x^2)^2-\cos(3-x^2)^2\right) \end{aligned}$$

e)
$$\begin{aligned} f'(x) &= \frac{\left(\sin(ax+b)+ax\cos(ax+b)\right)(x^2+3)-2x^2\sin(ax+b)}{(x^2+3)^2} \\ &= \frac{x^2\sin(ax+b)+3\sin(ax+b)+ax\cos(ax+b)\,(x^2+3)-2x^2\sin(ax+b)}{(x^2+3)^2} \\ &= \frac{(3-x^2)\sin(ax+b)}{(x^2+3)^2}+\frac{ax\cos(ax+b)}{x^2+3} \end{aligned}$$