

Aufgabe 12.30

Differenzieren Sie folgende Funktionen:

a) $y(x) = x^3 + 2x^2 - 4x + 13,$

b) $y(x) = \sqrt{x},$

c) $y(x) = x^5 - \frac{2}{x^2},$

d) $y(x) = (x^2 - 9)\sqrt{x},$

e) $y(x) = \frac{x}{x^2 + 5},$

f) $y(x) = \sqrt{x^2 + 64} \quad !$

Lösung:

a) $y'(x) = 3x^2 + 4x - 4$

b) $y(x) = x^{\frac{1}{2}}, \quad y'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

c) $y(x) = x^5 - 2x^{-2}, \quad y'(x) = 5x^4 + 4x^{-3} = 5x^4 + \frac{4}{x^3}$

d) Produktregel: $y'(x) = (x^2 - 9)' \sqrt{x} + (x^2 - 9)(\sqrt{x})' = 2x\sqrt{x} + (x^2 - 9) \left(\frac{1}{2\sqrt{x}} \right)$
 $= 2x\sqrt{x} + \frac{x\sqrt{x}}{2} - \frac{9}{2\sqrt{x}} = \frac{5}{2}x\sqrt{x} - \frac{9}{2\sqrt{x}}$

e) Quotientenregel: $y'(x) = \frac{x'(x^2 + 5) - x(x^2 + 5)'}{(x^2 + 5)^2} = \frac{x^2 + 5 - 2x^2}{(x^2 + 5)^2} = \frac{5 - x^2}{(x^2 + 5)^2}$

f) Kettenregel: $y = f(g(x)) = \sqrt{g(x)} = \sqrt{x^2 + 64}, \quad \frac{df}{dx} = \frac{df}{dg} \frac{dg}{dx}, \quad g(x) = x^2 + 64,$

$$y'(x) = \frac{d\sqrt{x^2 + 64}}{d(x^2 + 64)} \frac{d(x^2 + 64)}{dx} = \frac{1}{2\sqrt{x^2 + 64}} 2x = \frac{x}{\sqrt{x^2 + 64}} \quad (\text{zur Ableitung von } \sqrt{g} \text{ vgl. b)})$$