Aufgabe 12.6

Berechnen Sie ohne Verwendung der l'Hospitalschen Regel die Grenzwerte

a) $\lim\limits_{x\to-\infty} \frac{1-2x+3x^2-4x^3}{(1-2x)(4-3x^2)}$, b) $\lim\limits_{x\to 3} \frac{36-4x^2}{x^2+2x-15}$, c) $\lim\limits_{x\to-3} \left(\frac{1}{x+3}+\frac{11x+2}{2x^3+3x^2-5x+12}\right)$!

Lösung:

a) $$\lim_{x\to-\infty} \frac{1-2x+3x^2-4x^3}{(1-2x)(4-3x^2)} = \lim_{x\to-\infty} \frac{x^3\left(\frac{1}{x^3}-\frac{2}{x^2}+\frac{3}{x}-4\right)}{x\left(\frac{1}{x}-2\right)x^2\left(\frac{4}{x^2}-3\right)} = \lim_{x\to-\infty} \frac{\left(\frac{1}{x^3}-\frac{2}{x^2}+\frac{3}{x}-4\right)}{\left(\frac{1}{x}-2\right)\left(\frac{4}{x^2}-3\right)}$$
$$= \frac{0-0+0-4}{(0-2)(0-3)} = \underline{\underline{-\frac{2}{3}}}$$

b) $x^2+2x-15=0 \iff x_{1/2} = -1\pm\sqrt{1+15} = 3;-5$

$$\lim_{x\to 3} \frac{36-4x^2}{x^2+2x-15} = \lim_{x\to 3} \frac{(6-2x)(6+2x)}{(x-3)(x+5)} = \lim_{x\to 3} \frac{-2(x-3)(6+2x)}{(x-3)(x+5)} = \lim_{x\to 3} \frac{-2(6+2x)}{x+5} = \frac{-24}{8} = \underline{\underline{-3}}$$

c) Da für $x=-3$ die Beziehung $2x^3+3x^2-5x+12=0$ gilt, enthält der Nenner $2x^3+3x^2-5x+12$ den Faktor $x+3$.

$$(2x^3+3x^2-5x+12):(x+3) = 2x^2-3x+4$$
$$\begin{array}{l} \underline{2x^3+6x^2} \\ \quad -3x^2-5x+12 \\ \quad \underline{-3x^2-9x} \\ \qquad\quad 4x+12 \\ \qquad\quad \underline{4x+12} \\ \qquad\qquad\quad 0 \end{array}$$

$$\lim_{x\to-3} \left(\frac{1}{x+3}+\frac{11x+2}{2x^3+3x^2-5x+12}\right) = \lim_{x\to-3} \left(\frac{1}{x+3}+\frac{11x+2}{(x+3)(2x^2-3x+4)}\right)$$
$$= \lim_{x\to-3} \frac{2x^2-3x+4+11x+2}{(x+3)(2x^2-3x+4)} = \lim_{x\to-3} \frac{2x^2+8x+6}{(x+3)(2x^2-3x+4)} = \lim_{x\to-3} \frac{2(x^2+4x+3)}{(x+3)(2x^2-3x+4)}$$
$$= \lim_{x\to-3} \frac{2(x+1)(x+3)}{(x+3)(2x^2-3x+4)} = \lim_{x\to-3} \frac{2(x+1)}{2x^2-3x+4} = \frac{2\cdot(-2)}{31} = \underline{\underline{-\frac{4}{31}}}$$