Aufgabe 9.17

Berechnen Sie die Grenzwerte

a) $\lim\limits_{n\to\infty}\left(\frac{4n^4+n^3+n^2}{2n^2+n}-2n^2-1\right)$, b) $\lim\limits_{n\to\infty}\left(\frac{4n^4+2n^3+n^2}{2n^2+n}-2n^2-1\right)$,

c) $\lim\limits_{n\to\infty}\left(\frac{4n^4+2n^3+2n^2}{2n^2+n}-2n^2-1\right)$!

Lösung:

a) $$\lim_{n\to\infty}\left(\frac{4n^4+n^3+n^2}{2n^2+n}-2n^2-1\right)=\lim_{n\to\infty}\frac{4n^4+n^3+n^2-(2n^2+1)(2n^2+n)}{2n^2+n}$$
$$=\lim_{n\to\infty}\frac{4n^4+n^3+n^2-4n^4-2n^3-2n^2-n}{2n^2+n}=\lim_{n\to\infty}\frac{-n^3-n^2-n}{2n^2+n}$$
$$=\lim_{n\to\infty}\frac{n^3\left(-1-\frac{1}{n}-\frac{1}{n^2}\right)}{n^2\left(2+\frac{1}{n}\right)}=\lim_{n\to\infty}n\frac{-1-\frac{1}{n}-\frac{1}{n^2}}{2+\frac{1}{n}}=\lim_{n\to\infty}-\frac{n}{2}=\underline{\underline{-\infty}}$$

b) $$\lim_{n\to\infty}\left(\frac{4n^4+2n^3+n^2}{2n^2+n}-2n^2-1\right)=\lim_{n\to\infty}\frac{4n^4+2n^3+n^2-(2n^2+1)(2n^2+n)}{2n^2+n}$$
$$=\lim_{n\to\infty}\frac{4n^4+2n^3+n^2-4n^4-2n^3-2n^2-n}{2n^2+n}=\lim_{n\to\infty}\frac{-n^2-n}{2n^2+n}$$
$$=\lim_{n\to\infty}\frac{n^2\left(-1-\frac{1}{n}\right)}{n^2\left(2+\frac{1}{n}\right)}=\lim_{n\to\infty}\frac{-1-\frac{1}{n}}{2+\frac{1}{n}}=\underline{\underline{-\frac{1}{2}}}$$

c) $$\lim_{n\to\infty}\left(\frac{4n^4+2n^3+2n^2}{2n^2+n}-2n^2-1\right)=\lim_{n\to\infty}\frac{4n^4+2n^3+2n^2-(2n^2+1)(2n^2+n)}{2n^2+n}$$
$$=\lim_{n\to\infty}\frac{4n^4+2n^3+2n^2-4n^4-2n^3-2n^2-n}{2n^2+n}=\lim_{n\to\infty}\frac{-n}{2n^2+n}=\lim_{n\to\infty}\frac{1}{n}\,\frac{-1}{2+\frac{1}{n}}=\underline{\underline{0}}$$