

Aufgabe 6.170

Bestimmen Sie ein Polynom $P(x)$ höchstens 5-ten Grades, für welches die Beziehungen $P(1) = -2$, $P'(1) = -7$, $P''(1) = -14$, $P'''(1) = 24$, $P(2) = -4$, $P'(2) = 25$ gelten!

(Икрамов, Ch. D.: Russisch: Икрамов, Х. Д.: Задачник по линейной алгебре. Москва: Наука 1975. Aufgabe 4.5.51. S. 104)

Lösung:

$$\begin{array}{llll}
 P(x) & = a + bx + cx^2 + dx^3 + ex^4 + fx^5 & P(1) & = a + b + c + d + e + f & = -2 \\
 P'(x) & = b + 2cx + 3dx^2 + 4ex^3 + 5fx^4 & P'(1) & = b + 2c + 3d + 4e + 5f & = -7 \\
 P''(x) & = 2c + 6dx + 12ex^2 + 20fx^3 & P''(1) & = 2c + 6d + 12e + 20f & = -14 \\
 P'''(x) & = 6d + 24ex + 60fx^2 & P'''(1) & = 6d + 24e + 60f & = 24 \\
 & & P(2) & = a + 2b + 4c + 8d + 16e + 32f & = -4 \\
 & & P'(2) & = b + 4c + 12d + 32e + 80f & = 25
 \end{array}$$

1	1	1	1	1	1	-2	1	1	1	1	1	1	-2	1	1	1	1	0	0	0	0	0
1	2	4	8	16	32	-4	0	1	2	3	4	5	-7	0	1	2	3	0	0	0	0	-1
0	1	2	3	4	5	-7	0	0	1	3	6	10	-7	0	0	1	3	0	0	0	0	-3
0	1	4	12	32	80	25	0	0	0	1	5	16	12	0	0	0	1	0	0	0	0	0
0	0	2	6	12	20	-14	0	0	0	3	16	55	46	0	0	0	0	1	0	0	0	-4
0	0	0	6	24	60	24	0	0	0	1	4	10	4	0	0	0	0	0	0	1	0	2
1	1	1	1	1	1	-2	1	1	1	1	1	1	-2	1	1	1	0	0	0	0	0	0
0	1	3	7	15	31	-2	0	1	2	3	4	5	-7	0	1	2	0	0	0	0	0	-1
0	1	2	3	4	5	-7	0	0	1	3	6	10	-7	0	0	1	0	0	0	0	0	-3
0	1	4	12	32	80	25	0	0	0	1	4	10	4	0	0	0	1	0	0	0	0	0
0	0	1	3	6	10	-7	0	0	0	1	5	16	12	0	0	0	0	1	0	0	0	-4
0	0	0	1	4	10	4	0	0	0	3	16	55	46	0	0	0	0	0	0	1	0	2
1	1	1	1	1	1	-2	1	1	1	1	1	1	-2	1	1	0	0	0	0	0	0	3
0	1	2	3	4	5	-7	0	1	2	3	4	5	-7	0	1	0	0	0	0	0	0	5
0	1	3	7	15	31	-2	0	0	1	3	6	10	-7	0	0	1	0	0	0	0	0	-3
0	1	4	12	32	80	25	0	0	0	1	4	10	4	0	0	0	1	0	0	0	0	0
0	0	1	3	6	10	-7	0	0	0	0	1	6	8	0	0	0	0	1	0	0	0	-4
0	0	0	1	4	10	4	0	0	0	0	4	25	34	0	0	0	0	0	1	0	0	2
1	1	1	1	1	1	-2	1	1	1	1	1	1	-2	1	0	0	0	0	0	0	0	-2
0	1	2	3	4	5	-7	0	1	2	3	4	5	-7	0	1	0	0	0	0	0	0	5
0	0	1	4	11	26	5	0	0	1	3	6	10	-7	0	0	1	0	0	0	0	0	-3
0	0	2	9	28	75	32	0	0	0	1	4	10	4	0	0	0	1	0	0	0	0	0
0	0	1																				