

Aufgabe 6.123

Lösen Sie das lineare Gleichungssystem

$$\begin{aligned} x_1 + x_2 - 2x_3 - x_4 &= 9 \\ x_1 + 3x_2 + 2x_3 - 9x_4 &= 13 \\ 2x_1 - x_2 + x_3 - x_4 &= 1 \\ x_1 - 3x_2 - 10x_3 + 15x_4 &= 1 \\ -4x_1 + 9x_2 + x_3 - 15x_4 &= 23 \quad ! \end{aligned}$$

Lösung:

$$\begin{array}{cccc|c} 1 & 1 & -2 & -1 & 9 \\ 1 & 3 & 2 & -9 & 13 \\ 2 & -1 & 1 & -1 & 1 \\ 1 & -3 & -10 & 15 & 1 \\ -4 & 9 & 1 & -15 & 23 \\ \hline 1 & 1 & -2 & -1 & 9 \\ 0 & 2 & 4 & -8 & 4 \\ 0 & -3 & 5 & 1 & -17 \\ 0 & -4 & -8 & 16 & -8 \\ 0 & 13 & -7 & -19 & 59 \\ \hline 1 & 1 & -2 & -1 & 9 \\ 0 & 1 & 2 & -4 & 2 \\ 0 & -3 & 5 & 1 & -17 \\ 0 & -4 & -8 & 16 & -8 \\ 0 & 13 & -7 & -19 & 59 \end{array}$$

$$\begin{array}{cccc|c} 1 & 1 & -2 & -1 & 9 \\ 0 & 1 & 2 & -4 & 2 \\ 0 & 0 & 11 & -11 & -11 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -33 & 33 & 33 \\ \hline 1 & 1 & -2 & -1 & 9 \\ 0 & 1 & 2 & -4 & 2 \\ 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & -1 & -1 \\ \hline 1 & 1 & -2 & -1 & 9 \\ 0 & 1 & 2 & -4 & 2 \\ 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$

$$\begin{array}{cccc|c} 1 & 1 & 0 & -3 & 7 \\ 0 & 1 & 0 & -2 & 4 \\ 0 & 0 & 1 & -1 & -1 \\ \hline 1 & 0 & 0 & -1 & 3 \\ 0 & 1 & 0 & -2 & 4 \\ 0 & 0 & 1 & -1 & -1 \end{array}$$

$$\begin{aligned} x_1 &= 3 + x_4 \\ x_2 &= 4 + 2x_4 \\ x_3 &= -1 + x_4 \end{aligned}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ -1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ 1 \\ 1 \end{pmatrix}$$