

### Aufgabe 6.119

Lösen Sie das lineare Gleichungssystem

$$\begin{array}{rccccrcr} 4x_1 + 2x_2 & & & + 8x_4 - 4x_5 & & = & 12 \\ & 6x_2 + 2x_3 & & & + 2x_5 & = & 6 \\ & 9x_2 & - & x_4 + 3x_5 & & = & 4 \\ & 8x_2 & & & + 4x_5 - 2x_6 & = & 8 \end{array}$$

mit dem Gaußschen Algorithmus!

**Lösung:**

$$\begin{array}{cccccc|c} 4 & 2 & 0 & 8 & -4 & 0 & 12 \\ 0 & 6 & 2 & 0 & 2 & 0 & 6 \\ 0 & 9 & 0 & -1 & 3 & 0 & 4 \\ 0 & 8 & 0 & 0 & 4 & -2 & 8 \end{array} \quad \begin{array}{l} | \text{I}/4 \\ \\ \\ \end{array}$$

$$\begin{array}{cccccc|c} \mathbf{1} & \frac{1}{2} & 0 & 2 & -1 & 0 & 3 \\ 0 & 6 & 2 & 0 & 2 & 0 & 6 \\ 0 & 9 & 0 & -1 & 3 & 0 & 4 \\ 0 & 8 & 0 & 0 & 4 & -2 & 8 \end{array} \quad \begin{array}{l} \\ | \text{II}/2 \\ \\ \end{array}$$

$$\begin{array}{cccccc|c} 1 & \frac{1}{2} & 0 & 2 & -1 & 0 & 3 \\ 0 & 3 & \mathbf{1} & 0 & 1 & 0 & 3 \\ 0 & 9 & 0 & -1 & 3 & 0 & 4 \\ 0 & 8 & 0 & 0 & 4 & -2 & 8 \end{array} \quad \begin{array}{l} \\ \\ | \text{III}/(-1) \\ \end{array}$$

$$\begin{array}{cccccc|c} 1 & \frac{1}{2} & 0 & 2 & -1 & 0 & 3 \\ 0 & 3 & \mathbf{1} & 0 & 1 & 0 & 3 \\ 0 & -9 & 0 & \mathbf{1} & -3 & 0 & -4 \\ 0 & 8 & 0 & 0 & 4 & -2 & 8 \end{array} \quad \begin{array}{l} | \text{I} - 2\text{III} \\ \\ \\ \end{array}$$

$$\begin{array}{cccccc|c} 1 & \frac{37}{2} & 0 & \mathbf{0} & 5 & 0 & 11 \\ 0 & 3 & 1 & 0 & 1 & 0 & 3 \\ 0 & -9 & 0 & 1 & -3 & 0 & -4 \\ 0 & 8 & 0 & 0 & 4 & -2 & 8 \end{array} \quad \begin{array}{l} \\ \\ \\ | \text{IV}/(-2) \end{array}$$

$$\begin{array}{cccccc|c} 1 & \frac{37}{2} & 0 & 0 & 5 & 0 & 11 \\ 0 & 3 & 1 & 0 & 1 & 0 & 3 \\ 0 & -9 & 0 & 1 & -3 & 0 & -4 \\ 0 & -4 & 0 & 0 & -2 & \mathbf{1} & -4 \end{array} \quad \begin{array}{l} \\ \\ \\ \end{array} \quad \begin{array}{rccccrcr} x_1 + \frac{37}{2}x_2 & & & + 5x_5 & & = & 11 \\ & 3x_2 + x_3 & & + x_5 & & = & 3 \\ & -9x_2 & + x_4 - 3x_5 & & & = & -4 \\ & -4x_2 & & - 2x_5 - x_6 & & = & -4 \end{array}$$

$$\begin{aligned} x_1 &= 11 - \frac{37}{2}x_2 - 5x_5 \\ x_3 &= 3 - 3x_2 - x_5 \\ x_4 &= -4 + 9x_2 + 3x_5 \\ x_6 &= -4 + 4x_2 + 2x_5 \end{aligned}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} 11 \\ 0 \\ 3 \\ -4 \\ 0 \\ -4 \end{pmatrix} + s \begin{pmatrix} -\frac{37}{2} \\ 1 \\ -3 \\ 9 \\ 0 \\ 4 \end{pmatrix} + t \begin{pmatrix} -5 \\ 0 \\ -1 \\ 3 \\ 1 \\ 2 \end{pmatrix}, \quad s, t \text{ beliebig}$$