Aufgabe 6.95

Lösen Sie die Gleichungssysteme

a) $\begin{array}{c} 3x + 4y = 14 \\ -5x + 2y = 20 \end{array}$, b) $\begin{array}{c} 3x + 4y = 14 \\ -6x - 8y = 14 \end{array}$, c) $\begin{array}{c} 3x + 4y = 14 \\ -6x - 8y = -28 \end{array}$

grafisch und rechnerisch!
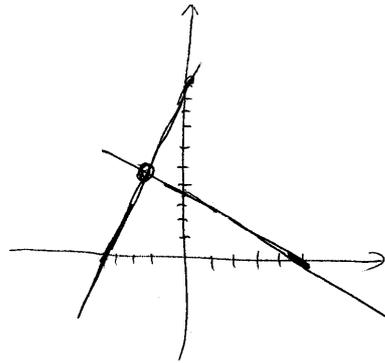
Lösung:

a) $3x + 4y = 14$

$y = 0 \;\rightarrow\; x = \dfrac{14}{3}$

$x = 0 \;\rightarrow\; y = \dfrac{14}{4} = \dfrac{7}{2}$

$-5x + 2y = 20$

$y = 0 \;\rightarrow\; x = -4$
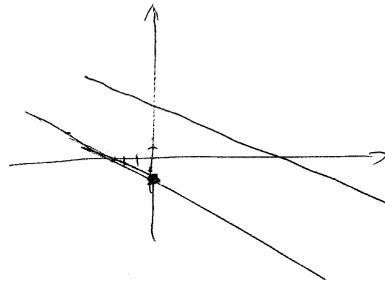
$x = 0 \;\rightarrow\; y = 10$

$$\begin{array}{rll} 3x + 4y = & 14 & + \\ -\;5x + 2y = & 20 & |\cdot 2 \\ -10x + 4y = & 40 & - \\ 13x \qquad\; = & -26 & \end{array}$$

$\underline{\underline{x = -2}},\; \underline{\underline{y}} = \dfrac{14 - 3\cdot(-2)}{4} \underline{\underline{\equiv 5}}$

b) $-6x - 8y = 14$

$y = 0 \;\rightarrow\; x = -\dfrac{14}{6} = -\dfrac{7}{3}$

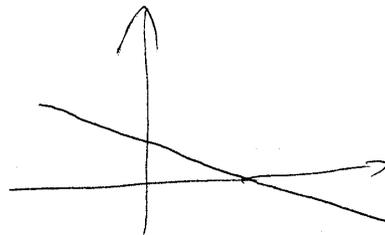$x = 0 \;\rightarrow\; y = -\dfrac{14}{8} = -\dfrac{7}{4}$

$$\begin{array}{ll} 3x + 4y = 14 & |\cdot 2 \\ -6x - 8y = 14 & + \\ 6x + 8y = 28 & + \\ \quad\; 0 = 42 & \end{array}$$

Widerspruch: GS unlösbar

(Geraden parallel)

c) $-6x - 8y = -28$

$y = 0 \;\rightarrow\; x = \dfrac{28}{6} = \dfrac{14}{3}$

$x = 0 \;\rightarrow\; y = \dfrac{28}{8} = -\dfrac{7}{2}$

$$\begin{array}{ll} 3x + 4y = \;\;14 & |\cdot 2 \\ -6x - 8y = -28 & + \\ 6x + 8y = \;\;28 & + \\ \quad\; 0 = 0 & \end{array}$$

immer erfüllt

(Geraden identisch)

Lösung des GS:

$x = t,\; y = \dfrac{14 - 3t}{4}$,

$t \in \mathbb{R}$, beliebig

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} t \\ \frac{7}{2} - \frac{3}{4}t \end{pmatrix}$

$= \begin{pmatrix} 0 \\ \frac{7}{2} \end{pmatrix} + t \begin{pmatrix} 1 \\ -\frac{3}{4} \end{pmatrix}$

$= \begin{pmatrix} 0 \\ \frac{7}{2} \end{pmatrix} + u \begin{pmatrix} 4 \\ -3 \end{pmatrix}$