

Aufgabe 5.63

Berechnen Sie $2^{-1000000} \left(\frac{5+3\sqrt{3}i}{4} - \frac{1}{1+\sqrt{3}i} \right)^{999999}$!

Lösung:

$$\frac{5+3\sqrt{3}i}{4} - \frac{1}{1+\sqrt{3}i} = \frac{5+3\sqrt{3}i}{4} - \frac{1-\sqrt{3}i}{(1+\sqrt{3}i)(1-\sqrt{3}i)} = \frac{5+3\sqrt{3}i}{4} - \frac{1-\sqrt{3}i}{4} = \frac{4+4\sqrt{3}i}{4} = 1+\sqrt{3}i$$

$$1+\sqrt{3}i = r(\cos \varphi + i \sin \varphi), \quad r = |1+\sqrt{3}i| = \sqrt{1+3} = 2, \quad \varphi = \arctan \frac{\sqrt{3}}{1} = \frac{\pi}{3} \text{ (da I. Quadrant)}$$

$$\begin{aligned} 2^{-1000000} \left(2 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right) \right)^{999999} &= \frac{2^{999999}}{2^{1000000}} \left(\cos \frac{999999\pi}{3} + i \sin \frac{999999\pi}{3} \right) \\ &= \frac{1}{2} (\cos \pi + i \sin \pi) = \underline{\underline{-\frac{1}{2}}} \end{aligned}$$