Aufgabe 5.51

Berechnen Sie $\dfrac{(\sqrt{3}+\mathrm{i})^{15}}{(1-\mathrm{i})^{22}}$!

Lösung:

$|\sqrt{3}+\mathrm{i}| = \sqrt{3+1} = 2,\quad \varphi = \arctan\dfrac{1}{\sqrt{3}} = \dfrac{\pi}{6}$ (da I. Quadrant), $\sqrt{3}+\mathrm{i} = 2\left(\cos\dfrac{\pi}{6}+\mathrm{i}\sin\dfrac{\pi}{6}\right)$

$1-\mathrm{i} = \sqrt{2}\left(\cos\left(-\dfrac{\pi}{4}\right)+\mathrm{i}\sin\left(-\dfrac{\pi}{4}\right)\right)$

$$\frac{(\sqrt{3}+\mathrm{i})^{15}}{(1-\mathrm{i})^{22}} = \frac{\left(2\left(\cos\frac{\pi}{6}+\mathrm{i}\sin\frac{\pi}{6}\right)\right)^{15}}{\left(\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right)+\mathrm{i}\sin\left(-\frac{\pi}{4}\right)\right)\right)^{22}} = \frac{2^{15}\left(\cos\frac{5\pi}{2}+\mathrm{i}\sin\frac{5\pi}{2}\right)}{2^{11}\left(\cos\left(-\frac{11\pi}{2}\right)+\mathrm{i}\sin\left(-\frac{11\pi}{2}\right)\right)}$$

$$= 2^{4}\left(\cos 8\pi+\mathrm{i}\sin 8\pi\right) = \underline{\underline{16}}$$