Aufgabe 5.47

Stellen Sie die folgenden komplexen Zahlen in der Form $a+b\mathrm{i}$ dar:

a) $\dfrac{(2\mathrm{i}+1)(\mathrm{i}-2)+1}{(2-\mathrm{i})^2-2+\mathrm{i}}$, b) $\left(\dfrac{1}{\sqrt{2}}(\sqrt{3}+\mathrm{i})\right)^{20}$, c) $\left(\dfrac{1}{\sqrt{2}}(\sqrt{3}+\mathrm{i})\right)^{30}$!

Lösung:

a) $\dfrac{(2\mathrm{i}+1)(\mathrm{i}-2)+1}{(2-\mathrm{i})^2-2+\mathrm{i}}=\dfrac{2\mathrm{i}^2+\mathrm{i}-4\mathrm{i}-2+1}{4-4\mathrm{i}+\mathrm{i}^2-2+\mathrm{i}}=\dfrac{-3-3\mathrm{i}}{1-3\mathrm{i}}=\dfrac{(-3-3\mathrm{i})(1+3\mathrm{i})}{(1-3\mathrm{i})(1+3\mathrm{i})}=\dfrac{6-12\mathrm{i}}{10}=\dfrac{3}{5}-\dfrac{6}{5}\mathrm{i}$

b) $|z|=\dfrac{1}{\sqrt{2}}\sqrt{3+1}=\sqrt{2},\ \varphi=\arctan\dfrac{1}{\sqrt{3}}=\dfrac{\pi}{6}$ (da I. Quadrant),

$z^{20}=\left(\sqrt{2}(\cos\dfrac{\pi}{6}+\mathrm{i}\sin\dfrac{\pi}{6})\right)^{20}=2^{10}(\cos\dfrac{20\pi}{6}+\mathrm{i}\sin\dfrac{20\pi}{6})=2^{10}(\cos\dfrac{4\pi}{3}+\mathrm{i}\sin\dfrac{4\pi}{3})=$

$2^{10}(-\dfrac{1}{2}-\mathrm{i}\dfrac{1}{2}\sqrt{3})=-2^9(1+\mathrm{i}\sqrt{3})=-512-512\sqrt{3}\mathrm{i}$

c) $|z|=\dfrac{1}{\sqrt{2}}\sqrt{3+1}=\sqrt{2},\ \varphi=\arctan\dfrac{1}{\sqrt{3}}=\dfrac{\pi}{6}$ (da I. Quadrant),

$z^{30}=\left(\sqrt{2}(\cos\dfrac{\pi}{6}+\mathrm{i}\sin\dfrac{\pi}{6})\right)^{30}=2^{15}(\cos\dfrac{30\pi}{6}+\mathrm{i}\sin\dfrac{30\pi}{6})=2^{15}(\cos 5\pi+\mathrm{i}\sin 5\pi)=$

$2^{15}(\cos\pi+\mathrm{i}\sin\pi)=-2^{15}=-32768$