Aufgabe 5.41

Sei a eine negative reelle Zahl und $z = a\sqrt{2+\sqrt{2}} + \mathrm{i}\,a\sqrt{2-\sqrt{2}}$. Geben Sie die Polar- und die exponentielle Darstellung von z an!

Lösung:

$z = a\sqrt{2+\sqrt{2}} + \mathrm{i}\,a\sqrt{2-\sqrt{2}}, \ \ |z| = |a|\sqrt{\sqrt{2+\sqrt{2}}^2 + \sqrt{2-\sqrt{2}}^2} = |a|\sqrt{2+\sqrt{2}+2-\sqrt{2}} = -2a$

$\tan\varphi = \dfrac{a\sqrt{2-\sqrt{2}}}{a\sqrt{2+\sqrt{2}}} = \dfrac{\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}} = \dfrac{(\sqrt{2-\sqrt{2}})(\sqrt{2-\sqrt{2}})}{(\sqrt{2+\sqrt{2}})(\sqrt{2-\sqrt{2}})} = \dfrac{2-\sqrt{2}}{\sqrt{4-2}} = \sqrt{2}-1 \approx 0{,}41421356,$

$\varphi = \arctan(\sqrt{2}-1) + 180° = 22{,}5° + 180° = 202{,}5° = \dfrac{9\pi}{8}$ \ \ (da III. Quadrant)

$z = -2a\,(\cos 202{,}5° + \mathrm{i}\,\sin 202{,}5°) = -2a\,\mathrm{e}^{\mathrm{i}\frac{9\pi}{8}}$