Aufgabe 1.19

Vereinfachen Sie folgende Ausdrücke:

a) $\dfrac{a^5-b^5}{b^5-a^5}$, b) $\left(\tan\dfrac{\pi}{3}-\tan\dfrac{\pi}{6}\right)\left(\cot\dfrac{\pi}{3}+\cot\dfrac{\pi}{6}\right)$, c) $\left(\tan\dfrac{\pi}{4}-\tan\dfrac{\pi}{6}\right)\left(\tan\dfrac{\pi}{4}-\tan\left(-\dfrac{\pi}{6}\right)\right)$,

d) $\sqrt[24]{x^{3^3}\Big/\left(x^{3^2}\,(x^3)^2\right)}$ e) $\dfrac{x^6-x^5+2x^4+10x^3-4x^2+16x+32}{x^3+2x+4}$, f) $\dfrac{(\tan^2 x)^3\,\cos^5 x}{\tan x\,\sin^4 x}$,

g) $\dfrac{1}{x}+\dfrac{2x}{x-2}-\dfrac{5}{x+3}-\dfrac{20}{x^2+x-6}-\dfrac{3}{x^2+3x}$, h) $\dfrac{(3\ln x+2\ln xy+\ln y)\ln\dfrac{1}{x}}{\ln\dfrac{1}{x^5y^3}}$!

Lösung:

a) $$\frac{a^5-b^5}{b^5-a^5}=\frac{a^5-b^5}{-(a^5-b^5)}=-1$$

b) $$\left(\tan\frac{\pi}{3}-\tan\frac{\pi}{6}\right)\left(\cot\frac{\pi}{3}+\cot\frac{\pi}{6}\right)=\left(\sqrt{3}-\frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}+\sqrt{3}\right)$$
$$=\left(\sqrt{3}-\frac{1}{\sqrt{3}}\right)\left(\sqrt{3}+\frac{1}{\sqrt{3}}\right)=3-\frac{1}{3}=\frac{8}{3}$$

c) $$\left(\tan\frac{\pi}{4}-\tan\frac{\pi}{6}\right)\left(\tan\frac{\pi}{4}-\tan\left(-\frac{\pi}{6}\right)\right)=\left(\tan\frac{\pi}{4}-\tan\frac{\pi}{6}\right)\left(\tan\frac{\pi}{4}+\tan\frac{\pi}{6}\right)$$
$$=\left(1-\frac{1}{\sqrt{3}}\right)\left(1+\frac{1}{\sqrt{3}}\right)=1-\frac{1}{3}=\frac{2}{3}$$

d) $$\sqrt[24]{x^{3^3}\Big/\left(x^{3^2}\,(x^3)^2\right)}=\sqrt[24]{x^{27}/(x^9x^6)}=\sqrt[24]{x^{12}}=\sqrt{|x|}$$

($\sqrt{x}$ ist als Ergebnis für negative x offensichtlich falsch!)

e) $(x^6-x^5+2x^4+10x^3-4x^2+16x+32):(x^3+2x+4)=x^3-x^2+8$

$$\begin{array}{l}
\underline{x^6\quad +2x^4+\ 4x^3}\\
\quad -x^5\qquad +\ 6x^3-4x^2+16x+32\\
\quad \underline{-x^5\qquad -\ 2x^3-4x^2}\\
\qquad\qquad\quad 8x^3\qquad +16x+32\\
\qquad\qquad\quad \underline{8x^3\qquad +16x+32}\\
\qquad\qquad\qquad\qquad\qquad\quad 0
\end{array}$$

f) $$\frac{(\tan^2 x)^3\,\cos^5 x}{\tan x\,\sin^4 x}=\frac{\tan^6 x\,\cos^5 x}{\tan x\,\sin^4 x}=\frac{\tan^5 x\,\cos^5 x}{\sin^4 x}=\frac{\sin^5 x\,\cos^5 x}{\cos^5 x\,\sin^4 x}=\sin x$$

g) $x^2+x-6=0$ für $x_1/2=-\dfrac{1}{2}\pm\sqrt{\dfrac{1}{4}+\dfrac{24}{4}}=-\dfrac{1}{2}\pm\dfrac{5}{2}=\begin{cases}\ \ 2\\ -3\end{cases}$, d.h. $x^2+x-6=(x-2)(x+3)$

$$\frac{1}{x}+\frac{2x}{x-2}-\frac{5}{x+3}-\frac{20}{x^2+x-6}-\frac{3}{x^2+3x}$$
$$=\frac{(x-2)(x+3)\ +\ 2x^2(x+3)\ -\ 5x(x-2)\ -\ 20x\ -\ 3(x-2)}{x(x-2)(x+3)}$$
$$=\frac{x^2+x-6+2x^3+6x^2-5x^2+10x-20x-3x+6}{x(x-2)(x+3)}=\frac{2x^3+2x^2-12x}{x(x-2)(x+3)}=\frac{2x(x-2)(x+3)}{x(x-2)(x+3)}=2$$

h) $$\frac{(3\ln x+2\ln xy+\ln y)\ln\frac{1}{x}}{\ln\frac{1}{x^5y^3}}=\frac{(\ln x^3+\ln x^2y^2+\ln y)\,(\ln 1-\ln x)}{\ln 1-\ln x^5y^3}=\frac{-\ln x^5y^3\ \cdot\ \ln x}{-\ln x^5y^3}=\ln x$$