Aufgabe 1.18

Vereinfachen Sie folgende Ausdrücke:

a) $\dfrac{a+b-c}{c-b-a}$, b) $\left(\sin\frac{\pi}{4}+\sin\frac{\pi}{3}\right)\left(\sin\frac{\pi}{4}+\sin\frac{\pi}{6}\right)\left(\cos\frac{\pi}{4}-\cos\frac{\pi}{3}\right)\left(\cos\frac{\pi}{4}-\cos\frac{\pi}{6}\right)$,

c) $\sqrt[11]{(x^2)^3\,x^{2^3}\,(-x)^{2^3}}$, d) $\dfrac{x^6+3x^5-9x^4+2x^3+30x^2-19x-56}{x^2+3x-8}$, e) $\dfrac{x^{1-n}\,x^{5n+7}}{(x^2)^{n+3}\,(x^n)^2}$,

f) $\dfrac{x-1}{x}+\dfrac{2}{x-1}+\dfrac{3x+1}{x-2}-\dfrac{1}{x^2-x}+\dfrac{3+x-3x^2}{x^2-3x+2}$, g) $\lg 800+3\lg\dfrac{1}{20}+\dfrac{\ln 153-\ln 17}{\ln 3}$!

Lösung:

a) $\dfrac{a+b-c}{c-b-a}=\dfrac{a+b-c}{-(a+b-c)}=-1$

b) $\left(\sin\frac{\pi}{4}+\sin\frac{\pi}{3}\right)\left(\sin\frac{\pi}{4}+\sin\frac{\pi}{6}\right)\left(\cos\frac{\pi}{4}-\cos\frac{\pi}{3}\right)\left(\cos\frac{\pi}{4}-\cos\frac{\pi}{6}\right)$

$=\left(\frac{1}{\sqrt2}+\frac{\sqrt3}{2}\right)\left(\frac{1}{\sqrt2}+\frac{1}{2}\right)\left(\frac{1}{\sqrt2}-\frac{1}{2}\right)\left(\frac{1}{\sqrt2}-\frac{\sqrt3}{2}\right)$

$=\left(\frac{1}{\sqrt2}+\frac{\sqrt3}{2}\right)\left(\frac{1}{\sqrt2}-\frac{\sqrt3}{2}\right)\left(\frac{1}{\sqrt2}+\frac{1}{2}\right)\left(\frac{1}{\sqrt2}-\frac{1}{2}\right)=\left(\frac12-\frac34\right)\left(\frac12-\frac14\right)=-\frac14\cdot\frac14=-\frac{1}{16}$

c) $\sqrt[11]{(x^2)^3\,x^{2^3}\,(-x)^{2^3}}=\sqrt[11]{x^6\,x^8\,x^8}=\sqrt[11]{x^{22}}=x^2$

d) $(x^6+3x^5-9x^4+2x^3+30x^2-19x-56):(x^2+3x-8)=x^4-x^2+5x+7$

$$\begin{array}{l}
\underline{x^6+3x^5-8x^4}\\
\quad -x^4+2x^3+30x^2-19x-56\\
\quad \underline{-x^4-3x^3+\ 8x^2}\\
\qquad 5x^3+22x^2-19x-56\\
\qquad \underline{5x^3+15x^2-40x}\\
\qquad\quad 7x^2+21x-56\\
\qquad\quad \underline{7x^2+21x-56}\\
\qquad\qquad 0
\end{array}$$

e) $\dfrac{x^{1-n}\,x^{5n+7}}{(x^2)^{n+3}\,(x^n)^2}=\dfrac{x^{1-n+5n+7}}{x^{2n+6+2n}}=\dfrac{x^{4n+8}}{x^{4n+6}}=x^2$

f) $x^2-3x+2=0$ für $x_1/2=\frac32\pm\sqrt{\frac94-\frac84}=\frac32\pm\frac12=\begin{cases}1\\2\end{cases}$, d.h. $x^2-3x+2=(x-1)(x-2)$

$\dfrac{x-1}{x}+\dfrac{2}{x-1}+\dfrac{3x+1}{x-2}-\dfrac{1}{x^2-x}+\dfrac{3+x-3x^2}{x^2-3x+2}$

$=\dfrac{(x-1)^2(x-2)\ +\ 2x(x-2)\ +\ (3x+1)x(x-1)\ -\ (x-2)\ +\ (3+x-3x^2)x}{x(x-1)(x-2)}$

$=\dfrac{(x^2-2x+1)(x-2)\ +\ 2x^2-4x\ +\ (3x^2+x)(x-1)\ -\ x+2\ +\ 3x+x^2-3x^3}{x(x-1)(x-2)}$

$=\dfrac{x^3-2x^2+x-2x^2+4x-2\ +\ 2x^2-4x\ +\ 3x^3+x^2-3x^2-x\ -\ x+2\ +\ 3x+x^2-3x^3}{x(x-1)(x-2)}$

$$= \frac{x^3-3x^2+2x}{x(x-1)(x-2)} = \frac{x(x^2-3x+2)}{x(x-1)(x-2)} = 1$$

g) $$\lg 800 + 3\lg\frac{1}{20} + \frac{\ln 153 - \ln 17}{\ln 3} = \lg 800 - 3\lg 20 + \frac{\ln\frac{153}{17}}{\ln 3} = \lg 800 - \lg 20^3 + \frac{\ln 9}{\ln 3}$$
$$= \lg\frac{800}{8000} + \frac{\ln 3^2}{\ln 3} = \lg\frac{1}{10} + \frac{2\ln 3}{\ln 3} = -1 + 2 = 1$$