Aufgabe 1.14

Vereinfachen Sie folgende Ausdrücke:

a) $\dfrac{x-y}{y-x}$, b) $\sqrt{(-x)^{-6}}$, c) $\dfrac{x^5+2x^4+6x^3+9x^2+19x+35}{x^2+2x+5}$,

d) $\dfrac{2-x}{4-x^2}+\dfrac{x+1}{x}-\dfrac{x+4}{x+2}-\dfrac{2}{x^2+2x}$, e) $\dfrac{(x^2)^4-x^{(2^4)}}{x^8}+x^8$, f) $\dfrac{x^{6n+2}x^{3-n}}{(x^2)^n(x^{n+3})^2}$!

Lösung:

a) $\dfrac{x-y}{y-x}=\dfrac{-(y-x)}{y-x}=-1$

b) $\sqrt{(-x)^{-6}}=\left(x^{-6}\right)^{\frac{1}{2}}=|x|^{-3}$

c) $(x^5+2x^4+6x^3+9x^2+19x+35):(x^2+2x+5)=x^3+x+7$

$$\begin{array}{l}
\underline{x^5+2x^4+5x^3} \\
\qquad\qquad x^3+9x^2+19x+35 \\
\qquad\qquad \underline{x^3+2x^2+\ \ 5x} \\
\qquad\qquad\qquad 7x^2+14x+35 \\
\qquad\qquad\qquad \underline{7x^2+14x+35} \\
\qquad\qquad\qquad\qquad\qquad 0
\end{array}$$

d) $\dfrac{2-x}{4-x^2}+\dfrac{x+1}{x}-\dfrac{x+4}{x+2}-\dfrac{2}{x^2+2x}=\dfrac{\cancel{2-x}\ 1}{\cancel{(2-x)}(2+x)}+\dfrac{x+1}{x}-\dfrac{x+4}{x+2}-\dfrac{2}{x^2+2x}$

$$=\frac{x+(x+1)(x+2)-x(x+4)-2}{x(x+2)}=\frac{x+x^2+3x+2-x^2-4x-2}{x(x+2)}=0$$

e) $\dfrac{(x^2)^4-x^{(2^4)}}{x^8}+x^8=\dfrac{x^8-x^{16}}{x^8}+x^8=1-x^8+x^8=1$

f) $\dfrac{x^{6n+2}x^{3-n}}{(x^2)^n(x^{n+3})^2}=\dfrac{x^{6n+2+3-n}}{x^{2n+2(n+3)}}=\dfrac{x^{5n+5}}{x^{4n+6}}=x^{5n+5-4n-6}=x^{n-1}$