Quantum models between order and disorder

In quantum mechanics the time evolution of a physical system is governed by the Schrödinger equation

$$\frac{d}{dt}u = -iHu;$$

here H – the Hamiltonian – is a selfadjoint operator in a Hilbert space. The spectral properties of H give important insight into the behavior of solutions to the Schrödinger equation. Typically, H is an unbounded partial differential operator. For one particle in an exterior field it takes the form $H = -\Delta + V$, where the negative Laplacian $-\Delta$ encodes the kinetic energy and V the potential energy. For general V it is impossible to determine the spectrum and its type. However there are cases in which at least certain qualitative features can be deduced:

If V vanishes near infinity (modeling a particle in the field of an ion) the spectrum has one part consisting of discrete energy levels and associated bound states and one part consisting of absolutely continuous spectrum associated with scattering states. This "atomic case" is extremely well studied and the predictions of the models are in astonishing agreement with the experimental data.

A simple model for ordered solids (like crystals) leads to potentials V that are periodic in the space coordinates. In this ordered case there is a technique called the Bloch or Floquet decomposition that gives spectral information. In this case one finds purely absolutely continuous spectra and no bound states.

These results can well be regarded as classical. A new part of the story began when P.W. Anderson explained the absence of effective transport in disordered media by using random potentials V and claiming that the associated Hamiltonians exhibit a transition from dense pure point to absolutely continuous spectra. The rigorous mathematical treatment of this so-called metal insulator transition is still missing.

The aim of this mini-course is an introduction to quantum Hamiltonians and their spectral properties. It will start with explaining the quantum mechanical background, include a discussion of spectral measures and spectral types and end with explaining recent progress and open questions in connection with random operators.

The course will consist of 3 lectures (of about 45-60 min) and probably 2 tutorials.