

Orthogonal polynomials and numerical methods for integral equations

The topics of this series of lectures and exercises are the following:

1. Basic properties of orthogonal polynomials
 - (a) Chebyshev and Jacobi polynomials
 - (b) Interpolation and quadrature processes based on the zeros of orthogonal polynomials
 - (c) Mapping properties (described by orthogonal polynomials) of singular and hypersingular integral operators
2. Stability of collocation methods for singular integral equations
 - (a) Basics from Functional Analysis
 - (b) A concept for the proof of the convergence of numerical methods for operator equations
 - (c) Application to collocation methods (based on the zeros of orthogonal polynomials) for integral equations
3. Design of fast algorithms
 - (a) Discussion of the structure of the discretized equations
 - (b) Two- or multiple grid methods
 - (c) Application of discrete trigonometric transforms and Krylov subspace methods

As examples we could consider the numerical solution of the generalized airfoil equation, of a nonlinear integral equation of Prandtl's type from fracture mechanics and of nonlinear Cauchy singular integral equations arising from certain two-dimensional free boundary value problems.

Amount: 3–5 lectures, 2–3 exercises (each 1–1,5 hour)