Initial value/boundary value problems for fractional diffusion-wave equations and inverse problems

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We consider initial value/boundary value problems for fraction diffusion-wave equation: $\partial_t^\alpha u(x,t) = Lu(x,t)$, where $0 < \alpha < 1$ or $1 < \alpha < 2$, and L is a uniformly elliptic operator with smooth cofficients depending on x. This is a partial differential equation with weakly singular integral: for $0 < \alpha < 1$, we define

$$\partial_t^\alpha u(x,t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-s)^{-\alpha} \frac{\partial u}{\partial s}(x,s)ds.$$

First we establish the unique existence of ths strong solutions and the asymptotic behaviour as the time t goes to ∞ and the proof is based on the eigenfunction expansions. Second for $\alpha \in (0,1)$, we apply the eigenfunction expansions and discuss some inverse problems:

(i) stability in the backward problem in time,

(ii) the uniqueness in determing an initial value, and

(iii) the uniqueness of solution by the decay rate as $t \to \infty$.

This is a joint work with Dr. Kenichi Sakamoto (University of Tokyo).