On some connections between pure Hankel operators, extensions of Helmholtz solutions into conical Riemann surfaces and the factorization of a special matrix

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Siegfried Prössdorf studied with me, in 1987-89, the factorization of Daniele-Khrapkov matrices in spaces of Hölder continuous functions. The work was motivated by the special matrix function

$$\Phi(\xi) = \begin{pmatrix} 1 & \zeta^{-1/2}(\xi) \\ -\zeta^{1/2}(\xi) & 1 \end{pmatrix}$$

where $\zeta(\xi) = \frac{\xi-k}{\xi+k}$, $\xi \in R$, $k = const$, $\Im m\, k > 0$. That matrix was found in diffraction theory by E. Meister in 1977 and "ad hoc factored" by A.R. Rawlins in 1981. It was recognized soon as an important example of a matrix function in $\mathcal{G}C^{\mu}(\ddot{R})^{2\times 2}$ that admits an explicit generalized factorization (although not rationally reducible to a triangular matrix), and gave a great impact on factorization theory, developed in our research center CEAF at Lisbon.

Recently diffraction by non-rectangular (but rational) wedges has shown an interesting connection with the existence of certain extension operators, either from half-lines in R^2 into cones bordered by this half-line on one side, or into cones which contain this half-line in its interior up to the common vertex, such that the extended function is a weak solution of the Helmholtz equation.

The existence of the latter extension operator is somehow equivalent to the inversion of a pure Hankel operator

$$H = r_+ A_{\zeta^{1/2}} J \ell_0 \quad : \quad L^2(R_+) \to L^2(R_+)$$

where $Jf(x) = f(-x)$ for $x \in R$, $A_{\zeta^{1/2}}$ is a convolution with Fourier symbol $\zeta^{1/2}$, r_+ denotes the restriction to the positive half-line and ℓ_0 the extension by zero to the full line.

In this lecture we focus on the relations between the above-mentioned operators and formulate some resulting open problems.

References

[1] S. Proessdorf and F.-O. Speck, A factorisation procedure for two by two matrix functions on the circle with two rationally independent entries, *Proc. Royal Soc. Edinburgh*, 115 A (1990), 119-138.