

# On some connections between pure Hankel operators, extensions of Helmholtz solutions into conical Riemann surfaces and the factorization of a special matrix

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Siegfried Prössdorf studied with me, in 1987-89, the factorization of Daniele-Khrapkov matrices in spaces of Hölder continuous functions. The work was motivated by the special matrix function

$$\Phi(\xi) = \begin{pmatrix} 1 & \zeta^{-1/2}(\xi) \\ -\zeta^{1/2}(\xi) & 1 \end{pmatrix}$$

where  $\zeta(\xi) = \frac{\xi-k}{\xi+k}$ ,  $\xi \in R$ ,  $k = \text{const}$ ,  $\Im m k > 0$ . That matrix was found in diffraction theory by E. Meister in 1977 and "ad hoc factored" by A.R. Rawlins in 1981. It was recognized soon as an important example of a matrix function in  $\mathcal{GC}^\mu(\ddot{R})^{2 \times 2}$  that admits an explicit generalized factorization (although not rationally reducible to a triangular matrix), and gave a great impact on factorization theory, developed in our research center CEAF at Lisbon.

Recently diffraction by non-rectangular (but rational) wedges has shown an interesting connection with the existence of certain extension operators, either from half-lines in  $R^2$  into cones bordered by this half-line on one side, or into cones which contain this half-line in its interior up to the common vertex, such that the extended function is a weak solution of the Helmholtz equation.

The existence of the latter extension operator is somehow equivalent to the inversion of a pure Hankel operator

$$H = r_+ A_{\zeta^{1/2}} J \ell_0 \quad : \quad L^2(R_+) \rightarrow L^2(R_+)$$

where  $Jf(x) = f(-x)$  for  $x \in R$ ,  $A_{\zeta^{1/2}}$  is a convolution with Fourier symbol  $\zeta^{1/2}$ ,  $r_+$  denotes the restriction to the positive half-line and  $\ell_0$  the extension by zero to the full line.

In this lecture we focus on the relations between the above-mentioned operators and formulate some resulting open problems.

## References

- [1] S. Proessdorf and F.-O. Speck, A factorisation procedure for two by two matrix functions on the circle with two rationally independent entries, *Proc. Royal Soc. Edinburgh*, 115 A (1990), 119-138.