From Toeplitz matrix-sequences to generalized locally Toeplitz sequences

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Recently, the class of generalized locally Toeplitz (GLT) sequences has been introduced as a generalization both of classical Toeplitz sequences and of variable coefficient differential operators. For every sequence of the class, a rigorous description of the asymptotic spectrum has been given in terms of a function (the symbol) that can be easily identified. The latter generalizes the notion of a symbol for differential operators (discrete and continuous) and for Toeplitz sequences where it is identified through the Fourier coefficients and is related to the classical Fourier Analysis. The GLT class has nice algebraic properties and indeed it has been proven that it is stable under linear combinations, products, and inversion when the sequence which is inverted shows a sparsely vanishing symbol (sparsely vanishing symbol = a symbol which vanishes at most in a set of zero Lebesgue measure). The GLT class is quite rich and virtually includes any Finite Difference or Finite Element discretization of PDEs, Toeplitz sequences with Lebesgue integrable symbols, and the algebra generated by such sequences. We will also present recent results in the non-Hermitian setting.