A Banach algebra framework for the finite sections of band-dominated operators

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For a given Banach space U and $1 < p < \infty$ let $l^p = l^p(\mathbb{Z}, U)$ denote the set of all p-summable sequences $(x_n)_{n\in\mathbb{Z}}$ of elements x_n in U. A band operator on l^p is a finite sum of the form $\sum a_k V_k$, where V_k is the usual shift by k positions and $a_k I$ denotes the operator of (entrywise) multiplication by the bounded function $a_k : \mathbb{Z} \to \mathcal{L}(U)$. The elements in the closure of the set of all band operators are referred to as band-dominated operators.

During the last years, the Fredholm properties of band-dominated operators and the stability of their finite section sequences have been extensively studied. A good compilation of the central questions and results can be found in the monographs [1] and [3]. Furthermore, a formula for the index of a Fredholm band-dominated operator A was derived in case $A = I + K$ with K having only compact entries in its matrix representation and under an additional condition on the space U (see [2]).

This talk will focus on the stability and Fredholm properties of sequences in the Banach algebra which is generated by the finite section sequences of band-dominated operators. Moreover, we will state an index formula for general band-dominated operators and arbitrary spaces U.

References

[1] M. Lindner, *Infinite Matrices and their Finite Sections* Birkhäuser Verlag, Basel, Boston, Berlin, 2006.

[2] V. Rabinovich, S. Roch, *The Fredholm index of locally compact band-dominated operators on $L^p(\mathbb{R})$* Integral Equations Operator Theory **57**(2007), no. 2, 263–281.

[3] V. Rabinovich, S. Roch, B. Silbermann, *Limit Operators and Their Applications in Operator Theory* Birkhäuser Verlag, Basel, Boston, Berlin, 2004.