Numerical solution of an inverse scattering problem by an elastic obstacle

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We consider the interaction between an elastic body and a compressible inviscid fluid, which occupies the unbounded exterior domain. The inverse problem of determining the shape of such an elastic scatterer from the measured far field pattern of the scattered fluid pressure field is of central importance in detecting and identifying submerged objects. For its numerical solution, we compare two different methods, both reducing the inverse problem to an optimization problem for the Fourier coefficients of a parametric representation for the boundary of the body.

First, we define the objective functional as the least squares deviation of the measured far-field data from the data generated by the finite-element solution of the direct fluid-solid interaction problem for the set of unknown Fourier coefficients. Surely, an additional regularization term is needed to cope with the ill-posedness of the problem. For the evaluation of this objective functional and its derivatives, a direct problem and the solution of a larger system of linear equations is required. In a second approach, we follow the ideas of Kirsch and Kress and represent the acoustic and elastic fields by potentials over auxiliary boundaries. Besides the above mentioned two terms in the objective functional, we have to add a penalty term forcing the potential fields to satisfy the transmission conditions over the approximate boundary of the scatterer. Though the number of unknown parameters increases by the degrees of freedom for the potential representation, the solution of the direct interaction problem is avoided. Moreover, the second method does not require any additional treatment of possible Jones modes. Recall that Jones modes are non-trivial solutions of the homogeneous direct problem for exceptional obstacles.

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References

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