On exponentially localized kernels based on Jacobi polynomials

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In this joint work with Frank Filbir and Hrushikesh N. Mhaskar we present exponentially localized kernels based on Jacobi polynomials and discuss their approximation power.

Let $\alpha, \beta \geq -1/2$, and for $k = 0, 1, \cdots$, ${p_k}^{(\alpha,\beta)}$ denote the orthonormalized Jacobi polynomial of degree k, we demonstrate the construction of a matrix H so that

$$\left| \sum_{k=0}^{\infty} H_{k,n} {p_k}^{(\alpha,\beta)}(\cos\theta) {p_k}^{(\alpha,\beta)}(\cos\phi) \right| \leq c_1 n^{2\max(\alpha,\beta)+2} \exp(-cn(\theta-\phi)^2),$$

where $\theta, \phi \in [0, \pi]$. Specializing to the case of Chebyshev polynomials, $\alpha = \beta = -1/2$, we apply this theory to obtain a construction of an exponentially localized polynomial basis for the corresponding Hilbert space $L^2(-1/2, -1/2)$.

The localization enables us also to obtain a characterization of local Besov spaces on the interval.