Fredholm integral equations on $[-1,1]$ with exponential weights

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The numerical treatment of Fredholm integral equations of the form

$$f(y) - \mu \int_{-1}^{1} k(x,y) f(x) v(x)\, \mathrm{d}x = g(y)\,, \qquad y \in [-1,1]\,,$$

where f is the unknown function, k and g are given functions, $\mu \in \mathbb{R}$ and v is a Jacobi weight, has been extensively studied in the literature.

Nevertheless, using Jacobi weights, we are bound to consider given functions which are continuous on $(-1,1)$ and can have algebraic singularities at ± 1. Therefore, in order to allow exponential increasing at ± 1, we consider a Pollaczek-type weight, defined by

$$w(x) = \mathrm{e}^{-(1-x^2)^{-\alpha}}\,, \qquad \alpha > 0\,.$$

Then we use a Nyström method for solving integral equations of the form

$$f(y) - \mu \int_{-1}^{1} k(x,y) f(x) w(x)\, \mathrm{d}x = g(y)\,, \qquad y \in [-1,1]\,,$$

where f is the unknown function, k and g are given functions, $\mu \in \mathbb{R}$ and w is a Pollaczek-type weight. We prove the stability and the convergence of the method and give a priori estimates of the error.

This talk concerns a joint work with G. Mastroianni.