Tensor-structured preconditioning of elliptic inverse in $\mathbb{R}^d$

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Modern methods of low rank tensor-product approximation allow efficient data-sparse representations of functions and operators in higher dimensions (cf. [1] – [7]). In particular, these methods apply to Green's functions given by classical volume potentials, to solution operators of elliptic/parabolic BVPs and stochastic PDEs, as well as to spectral projection operators arising from the Hartree-Fock equation in electronic structure calculations.

In this talk, we dicsuss the concept of rank structured truncated iteration for solving elliptic BVP/EVPs in $\mathbb{R}^d$. The approach is based on representation of all operators and functions in the low rank tensor product format to perform the preconditioned truncated iteration. We focus on the construction of low tensor rank spectrally close preconditioners and on the design of robust nonlinear tensor approximation in the Tucker/canonical formats in high dimension. The asymptotic complexity of the constructed tensor structured preconditioners is estimated by $\mathcal{O}(dn \log^* n)$, where n is the univariate discrete problem size. We present numerical illustrations on the efficiency of tensor methods applied to discrete problems on large spatial grids of size n^d.

References

[1] W. Hackbusch, B.N. Khoromskij, and E. Tyrtyshnikov: *Hierarchical Kronecker tensor-product approximation*, J. Numer. Math. **13** (2005), 119-156.

[2] B.N. Khoromskij: *An Introduction to Structured Tensor-Product Representation of Discrete Nonlocal Operators.* Lecture Notes **27**, MPI MIS, Leipzig 2005.

[3] B.N. Khoromskij: Structured Rank-$(r_1, ..., r_d)$ Decomposition of Function-related Tensors in $\mathbb{R}^d$. Comp. Meth. in Applied Math., **6** (2006), 2, 194-220.

[4] B.N. Khoromskij: *On Tensor Approximation of Green Iterations for Kohn-Sham Equations.* Comput. and Visual. in Sci., 11:259-271, DOI:10.1007/s00791-008-0097-x, 2008.

[5] B.N. Khoromskij: *Fast and Accurate Tensor Approximation of Multivariate Convolution with Linear Scaling in Dimension.* Preprint 36/2008, MPI MIS, Leipzig 2008 (J. Comp. Appl. Math., accepted).

[6] B.N. Khoromskij: *Tensor-structured Preconditioners and Approximate Inverse of Elliptic Operators in $\mathbb{R}^d$.* Preprint 82/2008, MPI MIS Leipzig, 2008 (J. Constructive Approx., accepted).

[7] B.N. Khoromskij and V. Khoromskaia: *Multigrid accelerated tensor approximation of function related multi-dimensional arrays.* Preprint 40, MPI MIS, Leipzig 2008 (submitted).