The fast multipole method and the connection problem for classical orthogonal polynomials

Jens Keiner
University of Lübeck, Germany

The Fast Multipole Method (FMM) is an important approximate algorithm, originally developed to accelerate the calculation of singular potentials for particle simulations. Since then, the method has seen numerous improvements and has been successfully used for similar problems, mostly in two- or three-dimensional domains.

A less known application is the use of the one-dimensional FMM to accelerate the calculation of eigendecompositions of certain structured matrices. Specifically, the FMM can here be used to apply the eigenvector matrix of an $n \times n$ matrix to any vector with $\mathcal{O}(n \log n \log(1/\varepsilon))$ arithmetic operations to any desired accuracy ε. Although the one-dimensional FMM is easier to implement than the higher-dimensional variants, careful modifications to the classical algorithm are necessary to ensure numerical stability.

A brief introduction to the modified one-dimensional FMM will be given and it will be shown how this can be used to speed up the computation of eigendecompositions of aforementioned structured matrices.

It is then possible to construct particular structured matrices, amenable to the FMM acceleration technique, whose eigenvector matrices contain the connection coefficients between classical orthogonal polynomials or their associated functions. This can be combined with above method to obtain a fast approximate algorithm to convert a finite expansion between those different sequences of polynomials and functions.

These matrices will be constructed explicitly for all classical orthogonal polynomials as well as their respective associated functions. A few numerical examples will be given.