An algorithm for the fast evaluation of the spherical mean value operator

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We consider the spherical mean value operator

$$M\colon L^2(\Omega) \to L^2(\partial\Omega, [0, \operatorname{diam}\Omega]), \quad Mf(x,r) = \frac{1}{\sigma(\mathbb{S}^{d-1})} \int_{\mathbb{S}^{d-1}} f(x + r\xi) \mathrm{d}\sigma(\xi),$$

which associates to a function f defined on some domain $\Omega \in \mathbb{R}^d$ its integrals along all arcs with center at the boundary of Ω. Our goal is to recover an approximation of the function f given a discrete sampling of its spherical means $Mf(x_l, r_k)$, $l = 1, \dots, L$, $k = 1, \dots, K$.

For this purpose we present a Fourier slice theorem for the spherical mean value operator, which we utilize for a fast algorithm for the computation of the forward algorithm. Now, a solution of the inverse problem can be found by applying CG methods on the normal equations.