On the approximation of the solutions of some elliptic boundary value problems by monogenic polynomial Appell sets

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Recently Appell systems of monogenic polynomials in $\mathbb{R}^3$ were constructed by several authors. Main purpose of the talk is to give a survey on the developments and to describe another Appell system that is complete in the space of square integrable quaternion-valued functions. A new Taylor type series expansion based on the Appell polynomials is presented which can be related to the corresponding Fourier series analogously as in the complex one-dimensional case. These results find applications in the description of the hypercomplex derivative as well as the monogenic primitive of a monogenic function.

The constructed Appell systems will be applied to the representation of solutions of boundary value problems. Based on a generalized Goursat theorem for the representation of biharmonic functions by two monogenic functions spatial (hypercomplex) Kolosov-Muskhelishvili formulas will be proved. Approximating the two monogenic functions in these representation formulas by functions from the monogenic Appell systems we obtain solutions of the Lame system. Some numerical examples will demonstrate the properties of the new approximate solutions.

The talk is based on joint work with S. Bock.