On quadratic summable solutions of refinement equations

Victor Didenko
Universiti Brunei Darussalam, Bandar Seri Begawan, Brunei

Refinement equations play an important role in wavelet analysis and computational mathematics. Discrete homogeneous refinement equations are used to construct wavelet bases, whereas discrete non-homogeneous equations arise while considering wavelets on compactly supported subsets of $\mathbb{R}^s$ – and also in signal processing to obtain multi-channel filters with good localization properties in time and frequency domains. On the other hand, continuous refinement equations have important applications in non-stationary subdivision processes, multiresolution analysis and wavelets, and invariant densities for model sets and quasicrystals.

Nevertheless, despite a steady growth of results available in literature, many issues remain open. The aim of this work is to study quadratic summable solutions of different types of homogeneous and non-homogeneous refinement equations from a unified point of view. Thus the properties established here do not depend on whether a refinement equation is discrete or continuous, scalar or vector – or whether univariate or multivariate case is considered. They rely upon properties of the corresponding dilation matrix; and if this matrix satisfies certain quite general conditions, then the solutions of the refinement equation have distinctive qualities. For example, for any homogeneous refinement equation, the set of non-trivial L_2-solutions is either empty or contains a set isomorphic to a space $L_\infty(\mathbb{V}_M)$ with a set $\mathbb{V}_M$ having a positive Lebesgue measure. Since the kernel of the adjoint operator possesses similar features, the corresponding refinement operator is Fredholm if and only if it is invertible.