Minimizing and maximizing the Euclidean norm of the product of two polynomials

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We investigate the problem of minimizing and maximizing the quotient

$$f_n(p,q) := \frac{\|pq\|}{\|p\|\,\|q\|},$$

where $p = p_0 + p_1x + \ldots + p_nx^n$ and $q = q_0 + q_1x + \ldots + q_nx^n$ are non-zero real or complex polynomials of maximum degree $n \in \mathbb{N}$ and

$$\|p\| := (|p_0|^2 + \ldots + |p_n|^2)^{\frac{1}{2}}$$

is simply the Euclidean norm of the polynomial coefficients. Clearly f_n is bounded and assumes its maximum and minimum value:

$$\min f_n = f_n(p_{\min}, q_{\min}), \quad \max f_n = f(p_{\max}, q_{\max}).$$

The squares of these values are overall sharp bounds for all eigenvalues of $(n+1)\times(n+1)$-autocorrelation Toeplitz matrices corresponding to arbitrary polynomials r of degree at most n with $\|r\| = 1$.