Weyl-type theorems for limit sets of the finite section spectrum

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Let A be a self-adjoint operator acting on a Hilbert space $\mathcal{H}$ densely defined in a domain $\mathrm{D}(A) \subseteq \mathcal{H}$. If B is A-compact, it is well-known that the essential spectra of A and $A+B$ coincide. Let $\mathcal{S} = (\mathcal{L}_n)$ be a sequence of finite dimensional subspaces such that $\mathcal{L}_n \subset \mathcal{L}_{n+1} \subset \mathrm{D}(A)$ and $\bigcup_{n=1}^{\infty} \mathcal{L}_n$ is dense in $\mathrm{D}(A)$ in the graph norm. Denote by A_n the compression of A to $\mathcal{L}_n$. The spectrum of A is a subset of the limit of the spectra of A_n,

$$\mathrm{Spec}(A) \subseteq \lim_{n\to\infty} \mathrm{Spec}(A_n),$$

but the later set might differ from the former in a non-trivial "polluted" set

$$\mathrm{Poll}(A, \mathcal{S}) = [\lim_{n\to\infty} \mathrm{Spec}(A_n)] \setminus \mathrm{Spec}(A).$$

In this talk we argue that $\mathrm{Poll}(A, \mathcal{S})$ has properties in common with the essential spectrum of A and discuss the following question: What sort of conditions on B, in the spirit of relative compactness, guarantee

$$\mathrm{Poll}(A, \mathcal{S}) = \mathrm{Poll}(A+B, \mathcal{S})?$$

The results announced are based on joint work with N. Boussaid and M. Lewin.