Smoothing Methods for Semidefinite Programs

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Outline

Notation

$$
\begin{aligned}
\mathcal{S}^{n\times n} &:= \{A \in \mathbb{R}^{n\times n} \mid A \text{ symmetric}\},\\
\mathcal{S}^{n\times n}_{+} &:= \{A \in \mathcal{S}^{n\times n} \mid A \text{ positive semidefinite}\},\\
\mathcal{S}^{n\times n}_{++} &:= \{A \in \mathcal{S}^{n\times n} \mid A \text{ positive definite}\},\\
A \succeq 0 &:\Longleftrightarrow A \in \mathcal{S}^{n\times n}_{+},\\
A \succ 0 &:\Longleftrightarrow A \in \mathcal{S}^{n\times n}_{++},\\
A \bullet B &:= \operatorname{trace}(AB^T) = \sum_{i,j=1}^{n} a_{ij} b_{ij} \quad \text{for } A, B \in \mathbb{R}^{n\times n},\\
A^{1/2} &:= \text{square root of } A \in \mathcal{S}^{n\times n}_{+},\\
e &:= (1, \ldots, 1)^T.
\end{aligned}
$$

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Semidefinite Programs

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Primal program (P):

$$\min\ C \bullet X \quad \text{s.t.} \quad A_i \bullet X = b_i \quad \forall i = 1, \ldots, m, \ \ X \succeq 0.$$

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Dual program (D):

$$\text{max} \;\; b^T \lambda \quad \text{s.t.} \quad \sum_{i=1}^{m} \lambda_i A_i + S = C, \;\; S \succeq 0.$$

Properties of Semidefinite Programs

The (primal or dual) semidefinite program is

- ▷ convex: $X_1, X_2 \succeq 0,\ \lambda \in (0,1)$
  $\Longrightarrow \lambda X_1 + (1-\lambda) X_2 \succeq 0$

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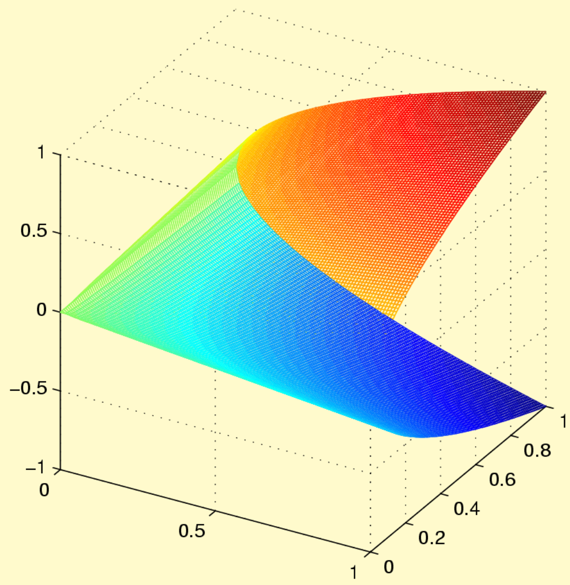
▷ nonsmooth: $X \succeq 0 \Longleftrightarrow \lambda_{\min}(X) \geq 0$

▷ nonpolyhedral: Example $n = 2$:

$$X = \begin{pmatrix} x & y \\ y & z \end{pmatrix} \succeq 0 \Longleftrightarrow x \geq 0, z \geq 0, xz \geq y^2$$



Optimality Conditions

Slater Constraint Qualification: $\exists\,(\hat{X}, \hat{\lambda}, \hat{S}) \in \mathcal{S}^{n\times n} \times \mathbb{R}^m \times \mathcal{S}^{n\times n}$ with

$$\sum_{i=1}^{m} \hat{\lambda}_i A_i + \hat{S} = C, \quad A_i \bullet \hat{X} = b_i \quad \forall i = 1, \ldots, m, \quad \hat{X} \succ 0, \; \hat{S} \succ 0.$$

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Then the following statements are equivalent:

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Central Path

Central path parameterized by $\tau^2 > 0$:

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\begin{aligned}
\sum_{i=1}^{m} \lambda_i A_i + S &= C, \\
A_i \bullet X &= b_i \ \forall i = 1, \ldots, m, \\
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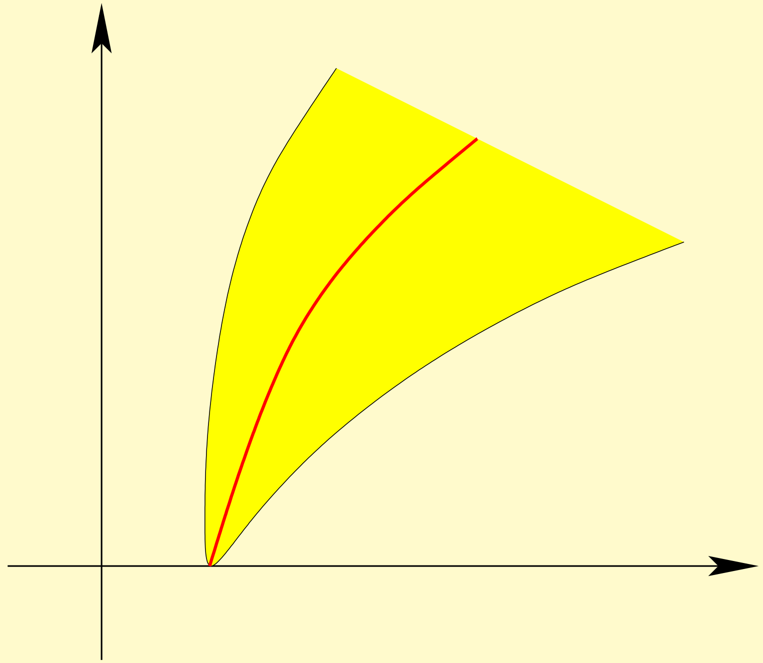
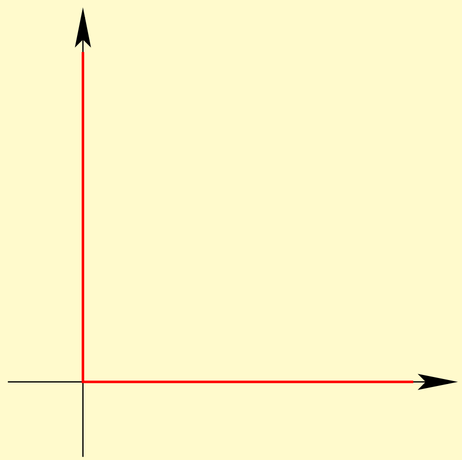


Interior-point methods typically solve a sequence of these problems (inexactly) with $\tau \longrightarrow 0$.

Reformulation

Let $\phi : \mathcal{S}^{n\times n} \times \mathcal{S}^{n\times n} \to \mathcal{S}^{n\times n}$ be any function having the following property:

$$\phi(X,S) = 0 \iff X \succeq 0, S \succeq 0, XS = 0.$$



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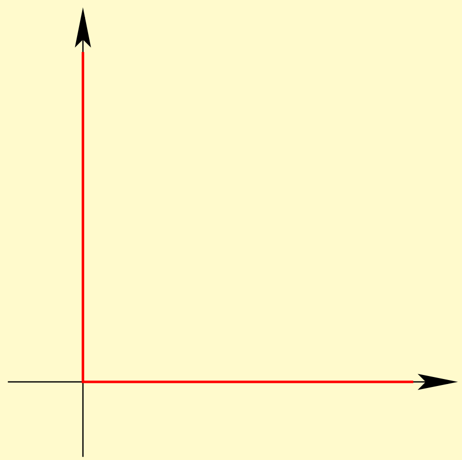

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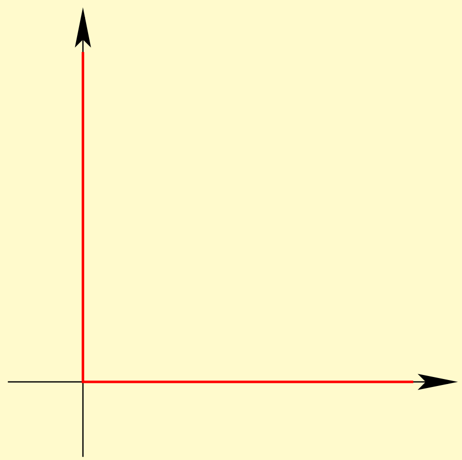

Examples for suitable functions ϕ:

- Fischer-Burmeister function: $\phi(X, S) := X + S - \left(X^2 + S^2\right)^{1/2}$;
- Minimum function: $\phi(X, S) := X + S - \left((X - S)^2\right)^{1/2}$.

Smoothing and Characterization of Central Path

The smoothed Fischer-Burmeister function

$$\phi(X, S, \tau) := X + S - \left(X^2 + S^2 + 2\tau^2 I\right)^{1/2}$$

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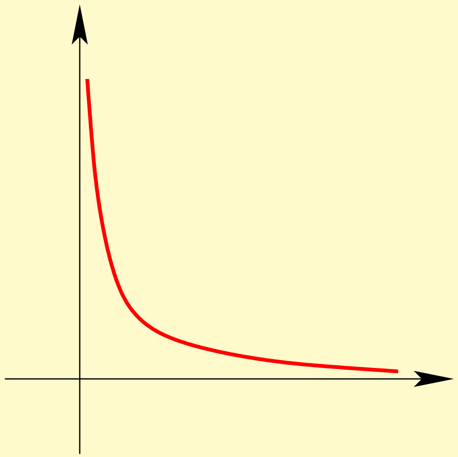

both have the following property:

$$X \succeq 0, S \succeq 0, XS = \tau^2 I \iff \phi(X,S,\tau) = 0.$$

Differentiability Properties

For $E \succ 0$ consider the Lyapunov operator

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THEOREM The smoothed minimum function is continuously differentiable with

$$\nabla \phi(X, S, \tau)(U, V, \mu) = U + V - L_E^{-1}\big[(X - S)(U - V) + (U - V)(X - S) + 8\tau\mu I\big]$$

whenever $E := \big((X - S)^2 + 4\tau^2\big)^{1/2} \succ 0$

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whenever $E := \big((X-S)^2 + 4\tau^2\big)^{1/2} \succ 0$ (similar for smoothed Fischer-Burmeister function).

Reformulation of Optimality Conditions

Optimality Conditions

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Reformulation as a nonlinear system of equations

$$\Phi(X, \lambda, S, \tau) = 0$$

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Reformulation as a nonlinear system of equations

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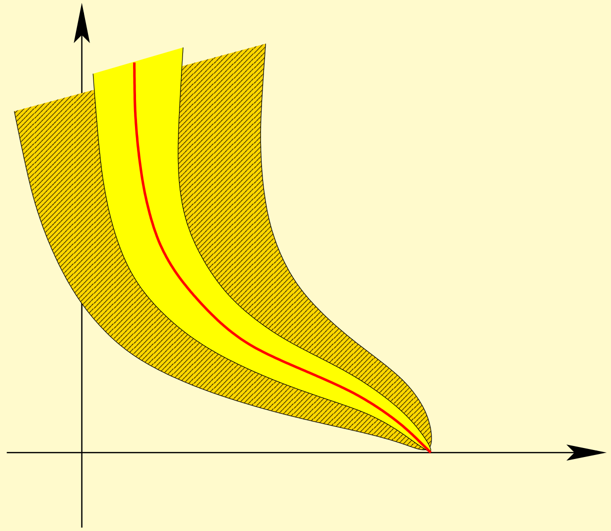
with

$$\Phi(X, \lambda, S, \tau) := \begin{pmatrix} \sum_{i=1}^{m} \lambda_i A_i + S - C \\ A_i \bullet X - b_i \quad (i = 1, \ldots, m) \\ \phi(X, S, \tau) \\ \tau \end{pmatrix}$$

Smoothing-type Method

Apply (a modification of) Newton's method to

$$\Phi(X, \lambda, S, \tau) = 0.$$



Then the Newton equation

$$\nabla\Phi(X, \lambda, S, \tau) \begin{pmatrix} \Delta X \\ \Delta\lambda \\ \Delta S \\ \Delta\tau \end{pmatrix} = -\Phi(X, \lambda, S, \tau)$$

has to be solved at each iteration.

Solution of Newton Equation

Decomposition of the Newton equations yields (for the smoothed minimum function) a linear system of equations

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Warning: When using the smoothed Fischer-Burmeister function, the corresponding matrix is positive definite, but not symmetric!

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- Assumptions:
  Linear independence of the matrices A_i and a new nondegeneracy condition, but no strict complementarity.

Three Nondegeneracy Conditions

Let (X^*, λ^*, S^*) be a solution of the optimality conditions. Nondegeneracy conditions:

- ▷ KSS nondegeneracy by Kojima, Shida, and Shindoh
- ▷ AHO nondegeneracy by Alizadeh, Haeberly, and Overton
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(c) KSS-nondegeneracy automatically implies strict complementarity.

(d) Under any of the three nondegeneracy conditions, the X^*- and S^*-components are unique.

Implementation

- ▷ MATLAB–Program with C MEX–Files
- ▷ Implementation based on SDPT^3-Solver by Todd, Toh and Tütüncü
- ▷ Main termination criterion:
$$\frac{\tau_k}{n} < 10^{-6}$$
- ▷ Starting point feasible with respect to the linear constraints

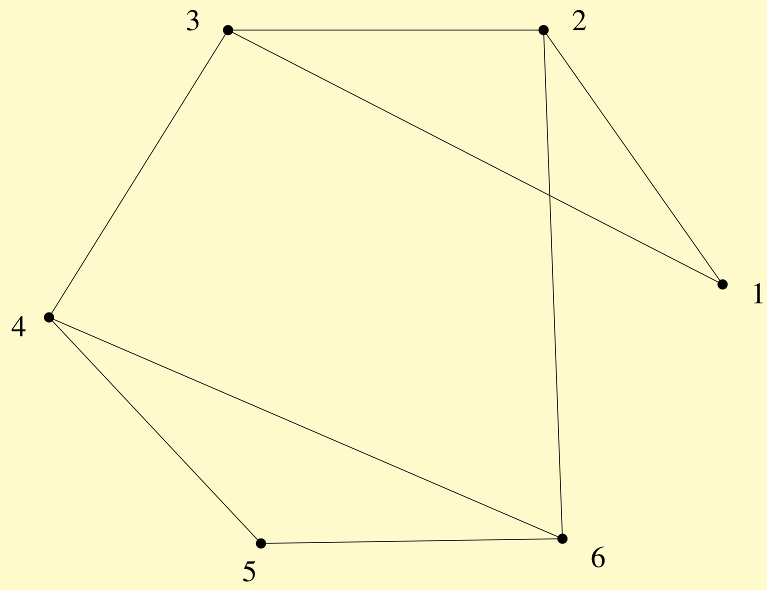
$$A_i \bullet X = b_i \quad \text{and} \quad \sum_{i=1}^{m} \lambda_i A_i + S = C$$

- ▷ Different test problems, including examples from SDPLIB

MAXCUT: A Bad Example

Consider MAXCUT with adjacency matrix

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$



MAXCUT: A Bad Example

Iteration history:

k	duality gap	$\|\phi(X, S, \tau)\|$
0	0.000000e–00	7.015009e+00
1	-7.518383e–01	4.788551e+00
2	3.627963e–02	2.595649e–01
3	-9.053267e–05	2.222778e–03
4	-2.172847e–09	2.092959e–07

Solution matrix X^*:

$$\begin{pmatrix} 1.0 & -0.5 & -0.5 & 0.5 & -1.0 & 0.5 \\ -0.5 & 1.0 & -0.5 & 0.5 & 0.5 & -1.0 \\ -0.5 & -0.5 & 1.0 & -1.0 & 0.5 & 0.5 \\ 0.5 & 0.5 & -1.0 & 1.0 & -0.5 & -0.5 \\ -1.0 & 0.5 & 0.5 & -0.5 & 1.0 & -0.5 \\ 0.5 & -1.0 & 0.5 & -0.5 & -0.5 & 1.0 \end{pmatrix}$$

Hence we may guess that

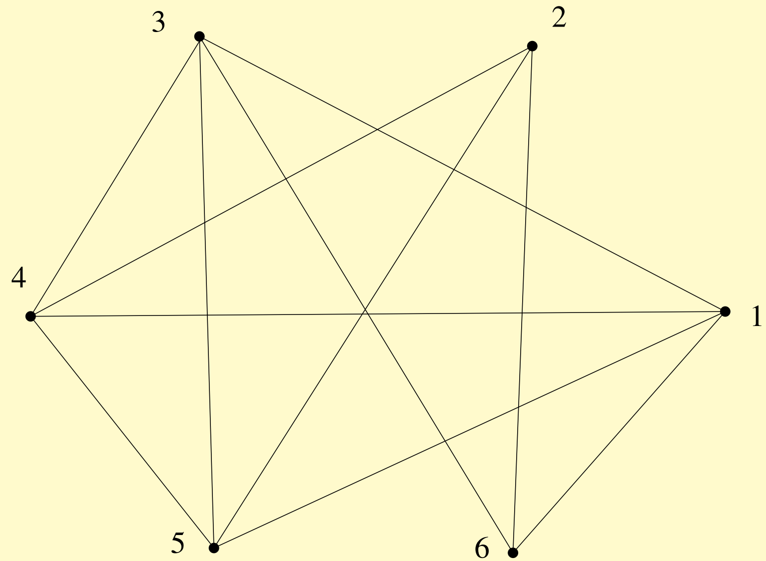
$$x := (1, -1, -1, 1, -1, 1)^T$$

is a solution of MAXCUT. But that's not true!

MAXCUT: A Good Example

Consider MAXCUT with adjacency matrix

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$



MAXCUT: A Good Example

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k	duality gap	$\|\phi(X, S, \tau)\|$
0	0.000000e–00	7.744872e+00
1	-6.824409e–01	4.581427e+00
2	3.361340e–02	2.484935e–01
3	-2.140298e–03	2.138911e–02
4	-6.860048e–06	6.180338e–05
5	-1.090483e–10	9.649522e–10

Solution matrix X^*:

$$\begin{pmatrix} 1 & 1 & 1 & -1 & -1 & -1 \\ 1 & 1 & 1 & -1 & -1 & -1 \\ 1 & 1 & 1 & -1 & -1 & -1 \\ -1 & -1 & -1 & 1 & 1 & 1 \\ -1 & -1 & -1 & 1 & 1 & 1 \\ -1 & -1 & -1 & 1 & 1 & 1 \end{pmatrix}$$

Hence we may guess that

$$x := (1, 1, 1, -1, -1, -1)^T$$

solves MAXCUT. That's true!

Average Iteration Numbers for Some SDPs of Small Dimension

Problem	n	m	AHO	HKM	NT	Min.
random	10	10	8.2	13.5	12.6	6.5
Norm min	20	6	8.0	9.3	10.1	6.7
Cheby	20	11	7.9	9.8	9.9	5.9
Maxcut	10	10	7.4	8.2	8.4	5.5
ETP	20	10	11.8	14.7	12.2	10.8
Lovasz	10	≈ 25	7.6	8.7	8.7	9.1
LogCheby	60	6	10.4	10.9	11.1	10.8
ChebyC	40	11	7.6	8.5	9.0	5.2

Average Iteration Numbers for Some SDPs of Medium Dimension

Problem	$\mathbf{n}$	$\mathbf{m}$	**AHO**	**HKM**	**NT**	**Min.**
random	20	20	10.2	14.4	13.1	8.9
Norm min	40	11	8.5	10.1	10.7	7.7
Cheby	40	21	7.7	9.7	10.0	6.1
Maxcut	21	21	8.2	9.6	9.6	6.3
ETP	40	20	12.6	16.7	13.3	14.0
Lovasz	21	$\approx$ 105	9.7	10.1	10.4	12.7
LogCheby	120	11	12.3	13.2	13.1	13.5
ChebyC	80	21	8.3	9.2	9.4	6.1

Average Iteration Numbers for Some SDPs of Larger Dimension

Problem	$\mathbf{n}$	$\mathbf{m}$	**AHO**	**HKM**	**NT**	**Min.**
random	50	50	10.4	15.6	13.7	10.9
Norm min	100	26	9.4	10.7	11.2	8.8
Cheby	100	27	9.3	10.4	11.4	7.1
Maxcut	50	50	9.0	10.0	10.5	6.7
ETP	100	50	13.7	18.4	15.1	19.1
Lovasz	30	$\approx$ 220	10.3	10.6	10.7	15.3
LogCheby	300	51	13.6	14.0	13.7	13.6
ChebyC	200	41	9.0	9.8	10.0	6.8

Some Examples from SDPLIB

Problem	n	m	AHO	NT	HKM	Min
mcp100	100	100	10	12	11	11
mcp124-1	124	124	10	14	12	28
mcp124-2	124	124	10	12	12	11
mcp124-3	124	124	11	12	12	9
mcp124-4	124	124	11	13	13	9
qap5	26	136	11	11	11	14
theta1	50	104	11	13	12	15
theta2	100	498	12	13	14	19
theta3	150	1106	12	14	14	18
theta4	200	1949	13	14	15	23
truss4	19	12	10	10	11	7
truss5	331	208	20	19	20	28
truss6	451	172	36	24	28	21
truss7	301	86	26	22	24	25
truss8	628	496	27	21	22	20

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