A Large Scale Stochastic Online Optimization Problem: Truck Scheduling for Inventory Management

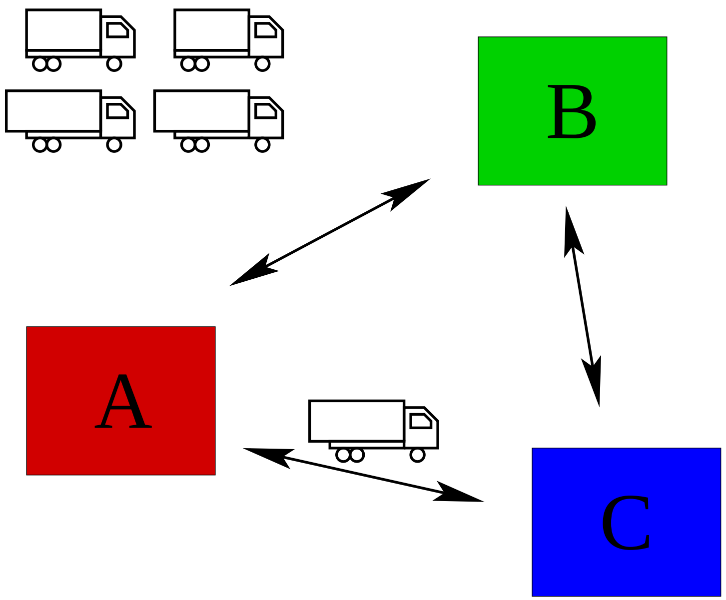
Christoph Helmberg
TU Chemnitz

Stefan Röhl
FH Vorarlberg
(project started at ZIB-Berlin)

- Overview
- Logistics
- Model
- Numerical Experiments

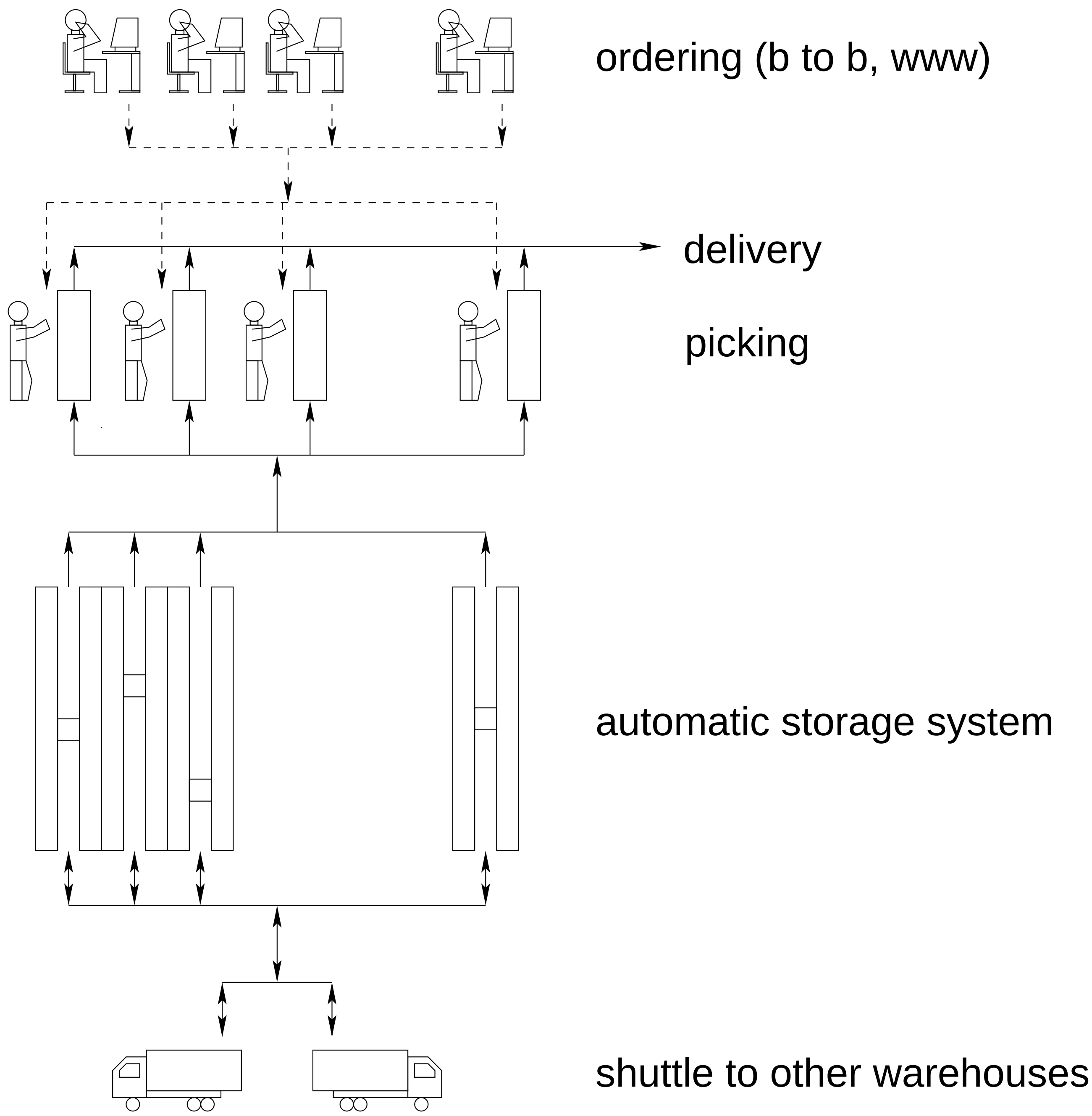
Overview

- eCom Logistics operates several warehouses within the same city
- logistics is centralized
- too many products (40000) to hold sufficient supply
  in each automatic storage system (50000-70000 pallets)
- trucks transport pallets between warehouses



- demand and supply shipments on short term
  but lots of data on past demand available

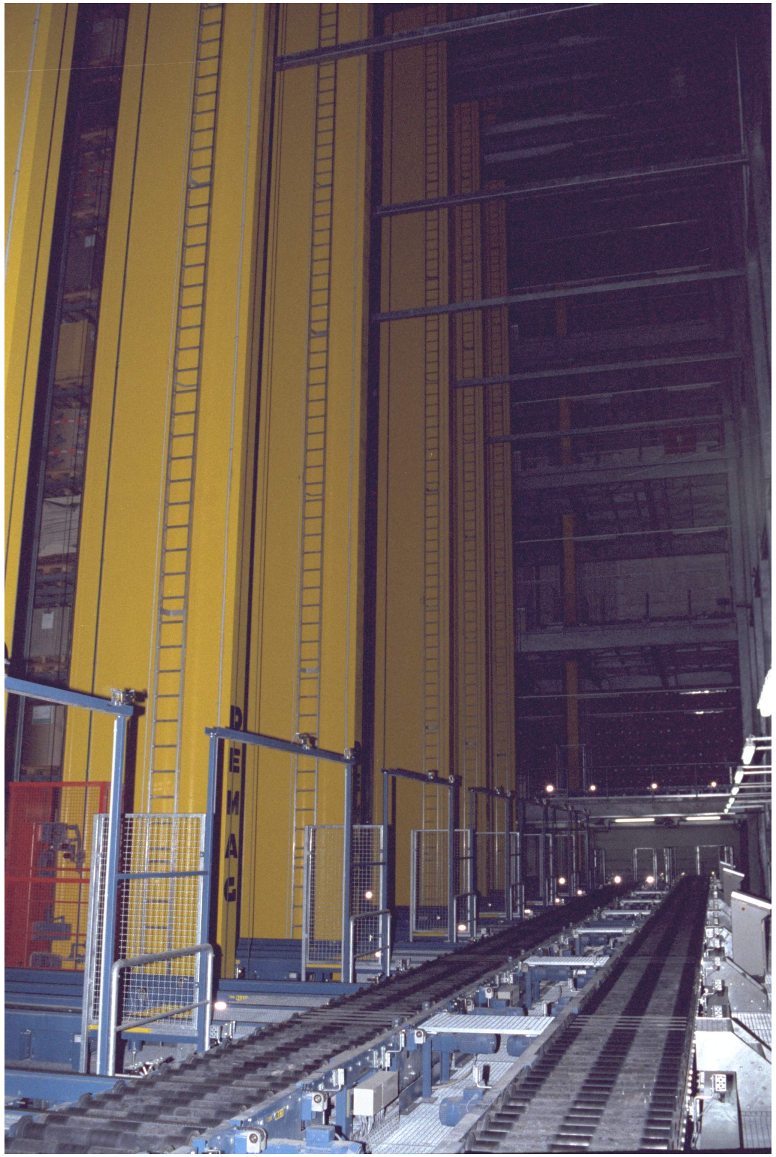
Goal: schedule the trucks, so that demand is satisfied on time


ordering (b to b, www)
delivery
picking
automatic storage system
shuttle to other warehouses

Pick to light

Greeting Cards

Automatic Storage System



- Delivery orders are accepted until 12:00 am,
  they are processed from 2:00 pm to 12:00 am on the next day.

- The required quantities should be available at the respective warehouses by 2:00 pm.

roughly 2000 deliveries and about 150000 delivery items per day

---

Pallets

- Between warehouses goods are transported on pallets (usually one type per pallet).

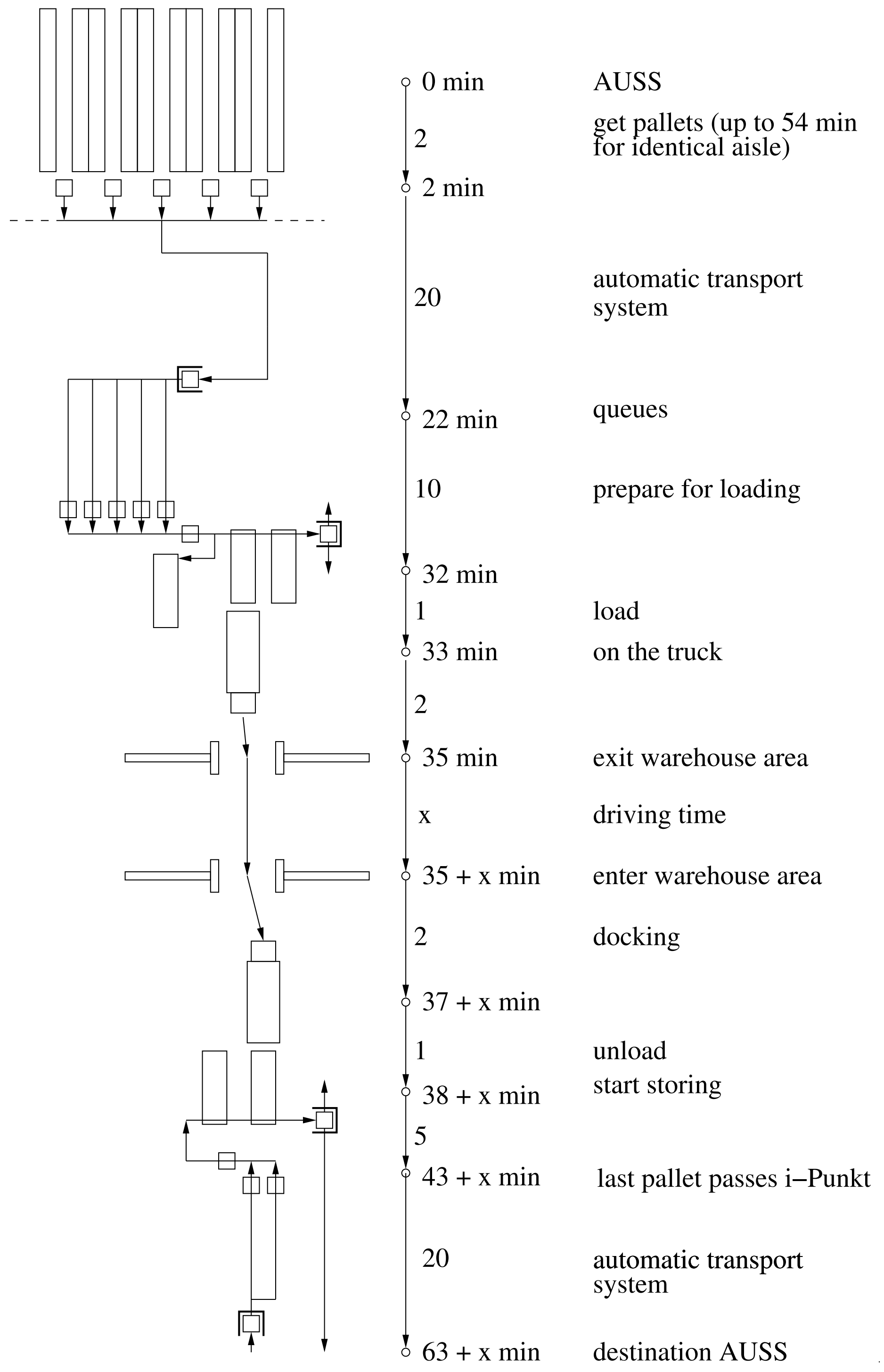
- Pallets vary in size during lifetime

→ storage requirement on trucks and in the automatic storage system (AUSS) not predictable

- Pallets may be retrieved by several agents, mostly by FIFO

→ cannot deal with particular pallets, but assume standard pallets (average number of items on a pallet known for most articles)

Trucks differ in capacity and operate on predefined hours,
they are rented several weeks in advance.


0 min
AUSS
2
get pallets (up to 54 min
for identical aisle)
2 min
20
automatic transport
system
22 min
queues
10
prepare for loading
32 min
1
load
33 min
on the truck
2
35 min
exit warehouse area
x
driving time
35 + x min
enter warehouse area
2
docking
37 + x min
1
unload
38 + x min
start storing
5
43 + x min
last pallet passes i–Punkt
20
automatic transport
system
63 + x min
destination AUSS

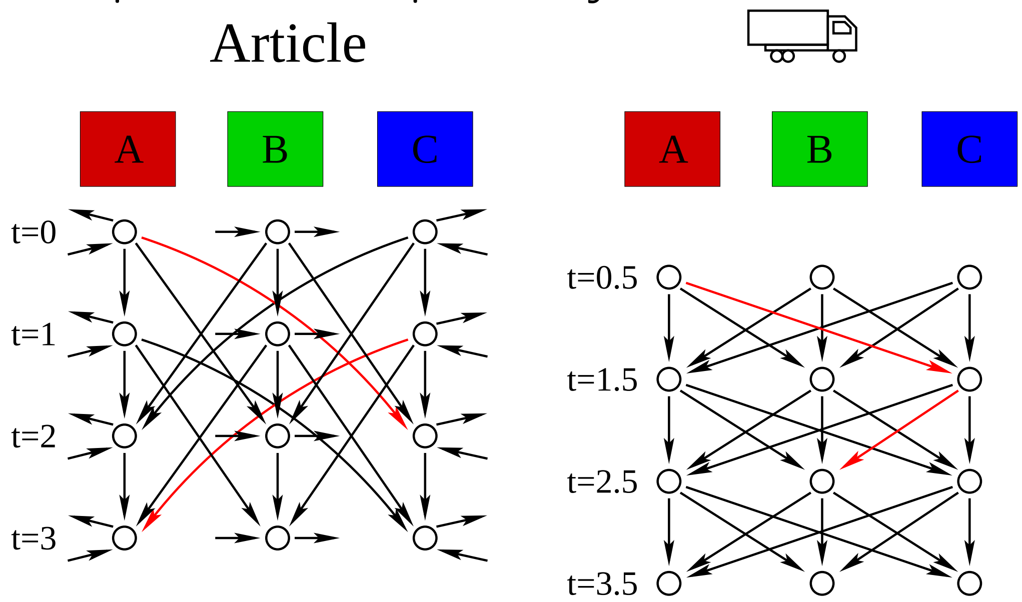
Model: Overview

For each article, pallets are assumed to have "standard measures".

Demand is calculated from a preliminary picking schedule plus some supply demand determined from statistical data.

For each article the flow of pallets between the warehouses is modeled by a network flow, discretized over time.

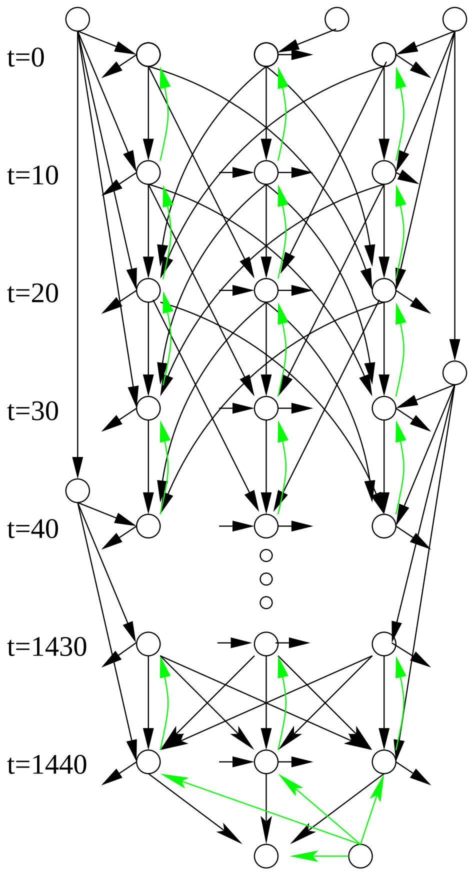
Capacity on transport arcs is opened by trucks that serve this arc.



Use Lagrangian relaxation on the coupling constraints.

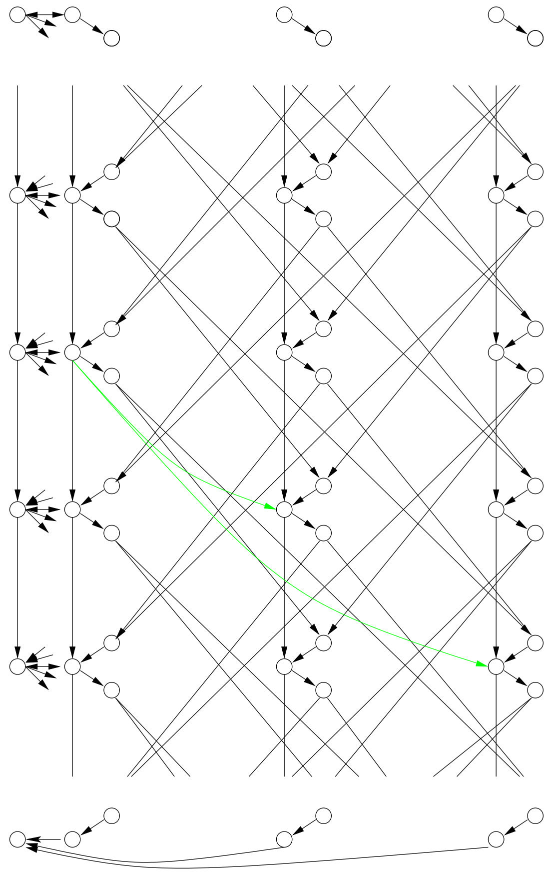
The Graphs (each step 10 min, one day in total)

for each article



transport buffer
infeasibility

for each truck



loading/unloading docks
transfers, depot

Balances and cost function?

Data

From an online input stream of all orders (with intended schedule), picks, movements of goods (10 MByte/day), we extract

- available stock
- pallets already scheduled for transport
- current demand
- distribution information on the demand

"Dirty Data"

- available amount may be negative (asynchronous system)
- some articles have standard pallet size = 0
- the current position of the trucks is unknown

Demand Distribution

Difficulty: Article IDs change each year → no long term data

Analyzing and exploiting correlations is out of scope even though demand for some products is clearly correlated

→ per article and warehouse:
empirical distribution based on the last twenty days.

Goals: (in this order)

1. transport prescheduled pallets as quickly as possible
   someone needs them now, they cannot be stopped and block the way
2. bring in pallets in time or even late to satisfy current demand
3. if there is still room, transport additional pallets so that chances for shortages are minimized but bring not more than are needed to provide supply for three days with given probability $\overline{\pi}$

Note: There are NO ACTUAL COSTS known for violations or failures

---

1 and 2 are ‘easy’:

1: additional graph for each direction of prescheduled pallets, put them into the buffer of the source warehouse and penalize their staying there

2: use balances in the networks, penalize infeasibility arcs slightly weaker

How to model 3?

General goal for 3:

- ‘with each successive transport
  maximize the minimum probability of survival for three days’

- No chance to model joint probability over all articles,

$\rightarrow$ use above empirical distribution for each article/warehouse,

$F_p^w(\alpha)$ ... probability that demand for p at w will be $\leq \alpha$

- simple linear approach: additional demand according to expectation does not reflect distribution well, hard to discern from demand

- stochastic programming: probability space enormous, how to construct scenarios?

---

we design a piecewise linear function that penalizes the lack of pallets according to the probability levels

- pallets should arrive ’sorted’ by their importance

if one of $p, \bar{p}$ fits on the next truck to w and

$F_p^w(\alpha_p) < F_{\bar{p}}^w(\alpha_{\bar{p}})$ then choose p

where α_p is the left over amount of p (can be reconstructed from flow)

Cost Function

If for an article the survival probability level is lowest,
supplying a pallet of this article should yield the greatest improvement

map probability to slope by function

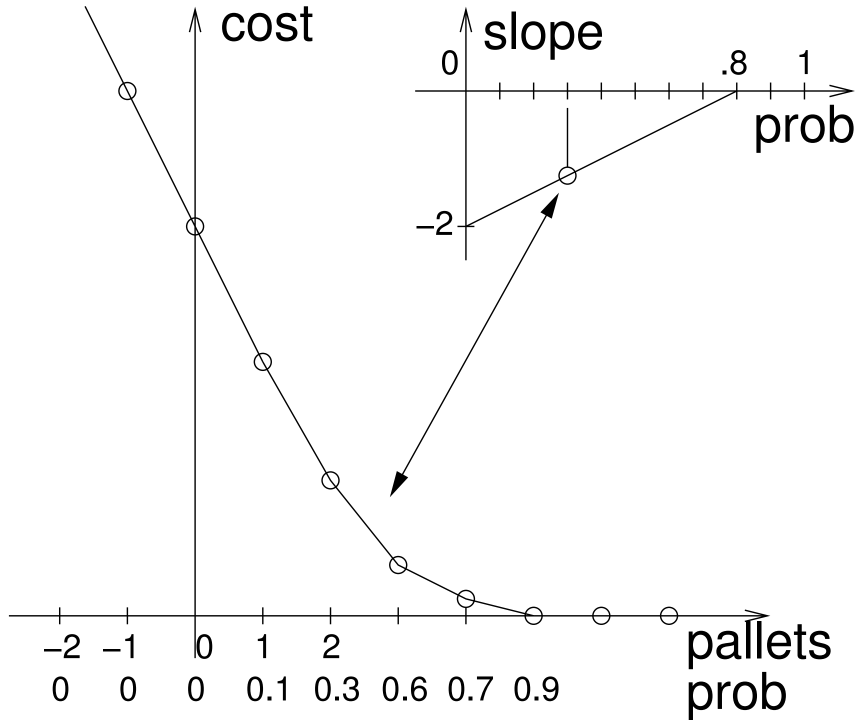
$$h : [0, 1] \to \mathbb{R}_-$$

strictly increasing on $[0, \overline{\pi}]$, 0 on $(\overline{\pi}, 1]$

where $0 < \overline{\pi} < 1$ is the goal level

Distribution can be given via a
map from #pallets to probability.

prob. is non decreasing with quantity
→ the cost function is convex



The slope at 0 should be matched to the infeasibility cost

First: Construct such a function for each article p and warehouse w
according to F_p^w for the flow remaining after the last time step

Theorem 1 *Let $h(x) = x - 1$, then the sum of all function values evaluated at the last time step yields the expected number of pallets, that have yet to be transported to cover the demand.*

$\rightarrow$ minimize expected number of pallets that still need transportation

---

Disadvantages:

1. transportation sequence of selected pallets does not matter
   $\rightarrow$ not useful if new schedule is computed half way (rolling horizon)

2. cost function prefers truck rides bringing many pallets with small gain to a ride with *one* important pallet
   $\rightarrow$ may not sufficiently reduce the probability to fail

---

approach to 1: use cost function at every quarter of the planning period (trade-off greedy/globally good/computation time)

approach to 2: choose an $h(x)$ that increases the gain for pallets that are needed with high probability

Choice of $h(x)$:

- h non increasing $\rightarrow$ cost function remains convex and piecewise linear
- Choose h identical for all p, w and strictly increasing
  $\rightarrow$ priority order among all pallets is preserved
- Are there candidates that can be motivated mathematically?

- Interpret $\quad \pi_p^w(j) = 1 - F_p^w(\alpha_p + j\theta_p) \quad$ as
  probability, that pallet j of article p is needed at w
- Assume that all these probabilities are independent (!!! never !!!), then

$$1 - \prod_{p,w,j} (1 - \pi_p^w(j))$$

is the probability that at least one pallet will be needed

$\rightarrow$ minimize $- \prod_{p,w,j} (1 - \pi_p^w(j)) = - \prod_{p,w,j} F_p^w(\alpha_p + j\theta_p)$

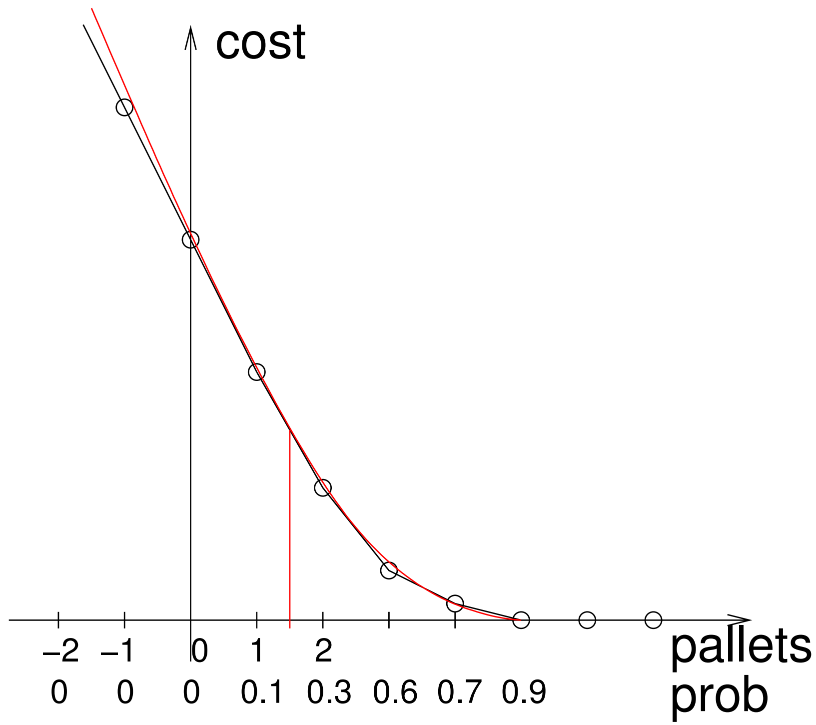
or equivalently, minimize $\sum_{p,w,j} - \log(F_p^w(\alpha_p + j\theta_p))$

$$h(x) = \begin{cases} -\log(\varepsilon) & 0 \le x \le \varepsilon \text{ for some fixed } \varepsilon > 0 \\ -\log(x) & \varepsilon < x < \bar{\pi} \\ 0 & \bar{\pi} \le x \le 1 \end{cases}$$

SOC Approximation to Cost Function

In order to avoid many inequalities, one could use SOC approximation

quadratic should have
- slope 0 at probability level $\bar{\pi}$
- same slope as cost function at some lower probability β



slacks s_i: available pallets $+s_i \geq$ # pallets at level α

SOC constraint: $s_0 \geq \sqrt{\sum h_i^2 s_i^2}$

$\rightarrow$ one joint cost term $c_0 s_0$ for all, asymptotic "linear" increase

Solution Method

Data is based on demand predictions, averaged pallet sizes and transport durations; solutions will not be realized in exactly this way but serve as a guideline only.

→ no need for optimal solutions, rough approximations suffice

---

- Use Lagrangian relaxation to decompose the problem into networks and convex functions.
- solve relaxation via a proximal bundle approach ($\sim$ fancy subgradient method)

⇒ yields a sequence of primal approximations that converge to a primal optimal solution [Feltenmark and Kiwiel 2000]

Sketch of Current Rounding Heuristic

- use path decomposition of primal solution to fix pallets that should be transported
- based on truck flows and priority orders within the pallets fix truck rides in a greedy manner starting from the beginning

---

Currently there is no improvement heuristic

Numerical Experiments

- Half a year of real data for two warehouses
  we constructed data for three warehouses by remapping articles

- 124 instances (=days)
- 200 – 1100 Articles need transportation, 800–1000 pallets
- one time step per 10 minutes → 144 steps
- 200,000 – 1,100,000 arcs, 1500 – 4500 multipliers (2WH)

MCF of A. Löbel and my ConicBundle Code on a Linux PC
(Pentium 4, 2.4 GHz, 1 GByte RAM [200-500 MB], Suse Linux 9.0)

- use a separate polyhedral model (2 cutting planes each) for each group
  - all article graphs
  - all truck graphs
  - all prescheduled pallet graphs
  - all piecewise linear cost functions (if used)

- use a separate nonpolyhedral model for SOC constraint (if used)

- 124 instances
- stop after 2000 evals or at subg. norm ≤ 5 and rel. precision ≤ 0.05

2 WH		p. lin.	[min,max]	SOC	[min,max]	LP	[min,max]
relax:	secs:	346	[5,618]	382	[2,2010]	4033.13	[36,100449]
	evals:	1697	[82,2001]	1718	[32,7557]		
	subg:	5.13	[1.97,11.7]	6.64	[1.44,41.4]		
	lpgap:	6.41	[0.57,23.4]				
heur:	secs:	134	[1,616]				
	gap:	8.94	[0.6,25.3]				
	lpgap:	2.72	[0.0,20.9]				

Simulation (prelim.): #pallets needed with certainty is reduced by 50%

---

3 WH		p. lin.	[min,max]	LP(92)	[min,max]
relax:	secs:	364	[7,713]	10497	[38,78556]
	evals:	1687	[151,2001]		
	subg:	4.78	[1.05,9.69]		
	lpgap:	7.75	[1.53,32.4]		
+ heur:	secs:	695	[11,2370]		
	gap:	23.6	[3.92,42.1]		
	lpgap:	17.4	[1.73,40.9]		

Conclusion

- approach seems to work very well, but there is room for improvement
- average time is ok, but without iteration limit variance is high,
  tested SOC approximations to piecewise linear functions (not better)
  simplex and interior point variants too expensive
  heuristic is easy to speed up a lot and should be easy to improve a bit
- make better use of data for generating distributions
- influence of uncertainty in pallet size, transportation time, etc.

- The practical quality of the solutions has yet to be evaluated.
- human control still needed because of some non automatic data
- How to prove ROI ???!!!

THANKS FOR COMING!