GAMM–SIAM Conference on Applied Linear Algebra Düsseldorf, July 24–27, 2006:

Mini-symposium: *Analysis and Stability of Second-Order Systems*

Peter Benner
TU Chemnitz
Fakultät für Mathematik
09107 Chemnitz
Germany
`benner@mathematik.tu-chemnitz.de`

Kresimir Veselic
FernUniversität Hagen
LG Mathematische Physik
P.O. Box 940
D-58084 Hagen
Germany
`kresimir.veselic@fernuni-hagen.de`

January 16, 2006

Second- and higher-order systems play a fundamental role in the modeling, simulation, control and optimization of mechanical systems and their interactions with other physical problems like microelectronics, fluid flow, and acoustics. Advances in mechatronics, microsystem technology and other concurrent research areas lead to new questions regarding the stability analysis of second-order systems. Moreover, the goal to control and optimize such systems also leads to new questions in systems and control theory. For decades, a standard approach was to re-formulate mechanical systems as first-order ordinary differential equations so that the related eigenproblems become classical linear or generalized eigenproblems. In recent years, more and more research results showed that this approach may lead to an unwanted increase of the sensitivity and condition of the resulting problems. Thus, new research directions both in theory and computing focus on the development of methods that address second-order systems directly. With the proposed mini-symposium we want to accommodate this research trend.

This mini-symposium aims at covering both theoretical and numerical aspects of second- (and higher-)order systems. Thus, the mini-symposium will have two parts, the first part being devoted to recent advances in analytical and theoretical aspects of second-order systems while the talks in the second part are devoted to new numerical methods for second-order systems.

The following talks will be given within this mini-symposium:

Part I: Theory

1. **Peter Lancaster** (University of Calgary, Canada):
   Inverse spectral problems for second-order systems
   Abstract: Recent work of the author and U. Prells on the expression of the three coefficients of a second order system in terms of spectral data (eigenvalues, eigen-

vectors and a sign characteristic) will be reviewed. Special attention is paid to the case of systems with symmetry properties.

2. **Christian Pommer** (Technical University of Denmark, Lyngby):
Gyroscopic stabilization and indefinite damped mechanical systems
Abstract: An important issue is how to modify a given unstable matrix in such a way that the resulting matrix is stable. We investigate in general under which condition a matrix $M + A$ is stable,where M is an arbitrary matrix and $A = -A^*$ is skew-Hermitian. We show that if trace(M) > 0 it is always possible to find a class of feasible skew-Hermitian matrices A depending on the choice of M. The theory can be applied to dynamical systems of the form

$$\ddot{x}(t) + (d\,D + g\,G)\,\dot{x}(t) + K\,x(t) = 0 \quad , \tag{1}$$

where $G = -G^T$ is a skew symmetric gyroscopic matrix, $D = D^T$ is a symmetric indefinite damping matrix and $K = K^T > 0$ is a positive definite stiffness matrix. d and g are scaling factors used to control the stability of the system. It is quite astonishing that when the damping matrix D is indefinite the system can under certain conditions be stable even if there are no gyroscopic forces G present. The Lyapunov matrix equation is used to predict the stability limit for pure dissipative systems as well as for dissipative systems with gyroscopic stabilization.

3. **Gottfried Spelsberg-Korspeter** (TU Darmstadt, Germany):
Stability and self excited vibrations of axially moving rods with frictional contact (with Peter Hagedorn, TU Darmstadt)
Abstract: Axially moving and vibrating elastic rods with bending stiffness (beams) occur frequently in technical applications. Examples are conveyor belts, band saws etc. In most cases the structural vibrations are unwanted and therefore possible excitation mechanisms have to be identified and suppressed by appropriate design.

This paper is devoted to the formulation and identification of a possible excitation mechanism for self excited vibrations in moving continua under frictional contact. The underlying model is a travelling beam sliding through two idealized brake pads. Using a Ritz discretization approach it is shown that self excited vibrations due to Coulomb friction occur resulting from an instability of the trivial solution of the linearized discretized equations of motion. Furthermore it is shown that the consistent formulation of the contact between the surface of the beam and the brake pads is essential for the stability behavior. The insights gained from the travelling beam also serve as an explanation for brake squeal and the methods used can also be applied to a rotating Kirchhoff plate.

4. **Emre Mengi** (New York University, USA):
A singular value characterization of the distance to uncontrollability for higher order systems and a practical algorithm exploiting the singular value characterization
Abstract: A robust measure of controllability of a dynamical system is its distance to the closest uncontrollable system. We generalize the distance to uncontrollability for linear systems to higher order systems and establish the equivalence of the distance to uncontrollability in the spectral norm for an mth order system to a singular value minimization problem over the complex plane. We provide a

practical algorithm for the higher order distance to uncontrollability exploiting the singular value characterization. The algorithm seeks imaginary eigenvalues of even-odd matrix polynomials and for a system of size n performs $O(m^4n^3/\arccos(1-(tol/m)^2))$ operations to compute an interval of length tol.

Part II: Numerical Methods

1. **Heinrich Voß** (TU Hamburg-Harburg, Germany):
   A local restart procedure for iterative projection methods for nonlinear symmetric eigenproblems
   (with Marta Markiewicz, TU Hamburg-Harburg)

   Abstract: For nonlinear eigenvalue problems $T(\lambda)x = 0$ satisfying a minmax characterization of its eigenvalues iterative projection methods combined with safeguarded iteration are suitable for computing all eigenvalues in a given interval. This approach hits it limitations if a large number of eigenvalues (in particular in the interior of the spectrum) is needed, since in this case one has to project the problem under consideration onto a sequence of search spaces of growing dimensions requiring an excessive amount of storage and computing time. In this paper we propose a local restart technique which projects the problem only to search spaces of limited dimension, and which is able to cope with this problem. An example of a gyroscopic eigenproblem demonstrates the efficiency of the new restart method.

2. **Christian Mehl** (TU Berlin, Germany):
   Triangular representations of second order systems
   (with Volker Mehrmann, TU Berlin)

   Abstract: The problem of solving second order systems leads to the quadratic eigenvalue problem $P(\lambda) = 0$, where $P(\lambda) = \lambda^2 M + \lambda C + k$ is a quadratic matrix polynomial. Usually, this problem is then solved by linearization.

   However, Gohberg, Kaashoek, and Lancaster observed that the so-called concept of *strong linearization* has to be used in order to properly reflect the structure at the eigenvalue infinity when the leading coefficient matrix M is singular. A corresponding equivalence relation (i.e., an equivalence relation that leaves the structure at the eigenvalue infinity invariant) can be defined on the set of all matrix polynomials.

   In this talk, we consider the question whether any quadratic matrix polynomial allows an equivalent representation, i.e., a representation with the same spectral information, where all coefficient matrices are upper triangular. Moreover, we discuss how the index of a second order system is reflected in the spectral information of the underlying matrix polynomial.

3. **Ninoslav Truhar** (University of Ossijek, Croatia):
   Bounds on the solution to the Lyapunov equation with a general stable matrix

   Abstract: We present some new estimates for the eigenvalue decay of the Lyapunov equation $A^*X + XA = B$ with a low rank right-hand side B. The new bounds show that the right-hand side B can greatly influence the eigenvalue decay rate of the solution. This suggests a new choice of the ADI-parameters for the iterative solution. The advantage of these new parameters is illustrated on second order damped systems with a low rank damping matrix. We will also present the new perturbation bound for the solution X of the Lyapunov equation with the general matrix A.

4. **Martin Stoll** (University of Oxford, United Kingdom):
A structured-exploiting Krylov-Schur-type method for second-order eigenproblems with Hamiltonian symmetry
(with Peter Benner, TU Chemnitz)

Abstract: Second-order eigenproblems with Hamiltonian spectral symmetry arise, for instance, from the stability analysis of gyroscopic systems or acoustic fluid-structure interaction. We show how a Krylov-Schur type algorithm based on the symplectic Lanczos process can be applied to such problems. The Krylov-Schur approach facilitates deflation, locking and purging strategies for the implicitly restarted symplectic Lanczos process originally introduced by Benner and Faßbender in 1997. We demonstrate the efficency of the method for several eigenproblems arising from Finite Element models in the application context mentioned above.