Universität Mannheim
Lehrstuhl für Mathematik VI

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# Exercises Algebraic Geometry
# Sheet 2

1. (a) Let $X$ be a topological space. Show that $\dim(X) = 0$ if $X$ is Hausdorff.
   (b) Show that the Zariski topology on $k^n$ is not Hausdorff for $n > 0$.
   (c) Show that any open set of an irreducible topological space is a dense subset.

2. Let $k$ be an algebraically closed field, $A$ be a finitely generated $k$-algebra and $I \subset A$ an ideal. Show that
$$\sqrt{I} = \bigcap_{\substack{\mathbf{m} \supset I \\ \mathbf{m} \text{ maximal}}} \mathbf{m}$$

3. (a) Compute the radical of the following ideals.
      i. $(x^p, y^q) \subset k[x, y]$
      ii. $(x^2y^3, z) \subset k[x, y, z]$
      iii. $(xy, x^2) \subset k[x, y]$
   (b) Show that for any algebraic sets $X_1, X_2 \subset k^n$, we have $I(X_1 \cap X_2) = \sqrt{I(X_1) + I(X_2)}$ and find an example where $I(X_1 \cap X_2) \neq I(X_1) + I(X_2)$. Try to understand the geometric reason for this inequality.

4. Find the decomposition into irreducible components of the following algebraic sets.
   (a)
   $$V(x^n + a_1x^{n-1} + \ldots + a_n) \subset \mathbb{C}$$
   for any $a_1, \ldots, a_n \in \mathbb{C}$.
   (b)
   $$X = V(x^2 + y^2 - 1, y^2 - x^3 - 1) \subset \mathbb{C}^2$$
   (c)
   $$X = V(x^2 - yz, xz - x) \subset \mathbb{C}^3$$

Alle Informationen zur Vorlesung (Termine, Übungsblätter, Skript etc.) sind unter

`http://hilbert.math.uni-mannheim.de/~sevenhec/AlgGeom07.html`

zu finden.