Periodic multiresolution

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One of the basic ideas of multiresolution and wavelet analysis consists in the investigation of shift-invariant function spaces. In this talk one-dimensional shift invariant spaces of periodic functions are revisited and generalized to multivariate shift-invariant spaces on non-tensor-product patterns. Here, we discuss matrix shifts, where M is a $(d \times d)$-integer matrix with det $M > 1$. Accordingly, an element of the shift-invariant space spanned by a function φ is given as the linear combination

$$\sum_{\ell \in \Gamma} c_\ell \varphi(\cdot - 2\pi M^{-1}\ell),$$

where Γ denotes the full collection of coset representatives of $Z^d/\Gamma Z^d$. Decompositions of shift invariant spaces are given by divisibility considerations.

For these spaces we discuss the dimension and we construct interpolatory and orthonormal bases. Possible patterns are classified. The results are applied to construct multivariate orthogonal Dirichlet kernels and the respective wavelets.

Moreover, we present some ideas of time-frequency-localization for trigonometric polynomial spaces. Particularly we will focus on uncertainty principles for periodic functions.