Topics in Singular Integrals

A. Böttcher

I will give two lectures. The first is

Markov inequalities, norms of Volterra operators, and zeros of Bessel functions

and concerns the problem of estimating the norm of a higher derivative of a polynomial in terms of the norm of the polynomial itself. This topic has a fascinating history, dating back to Mendeleev, best known for the creation of the periodic table of elements. Eventually it leads to determining the norms of large Toeplitz matrices, which may be viewed as the elementary building blocks of singular integral operators, and these norms are shown to be related to the norms of certain Volterra integral operators. Entering the last topic allows me to report on another captivating story ...

The second lecture, entitled

Two projections and the beauties of the Cauchy singular integral,

is devoted to the intriguing interplay between two projections theorems and the spectral theory of singular integral operators. I give an introduction to the game with two projections and then show how by employing the two projections strategy one can reduce the spectral theory of general singular integral operators with piecewise continuous coefficients to the identification of the spectra of certain local representatives. This identification is a great mathematical challenge, but also an extremely thrilling matter. You will enjoy lots of very nice pictures illustrating the beautiful metamorphosis of local spectra of singular integral operators from circular arcs to quite exotic but completely understood sets.